Proof Complexity of Long-Distance Q-Resolution

Tomáš Peitl, Friedrich Slivovsky, and Stefan Szeider

TU WIEN

ac ALGORITHMS AND COMPLEXITY GROUP

Proof Complexity of Long-Distance Q-Resolution

Fragments of

Tomáš Peitl, Friedrich Slivovsky, and Stefan Szeider

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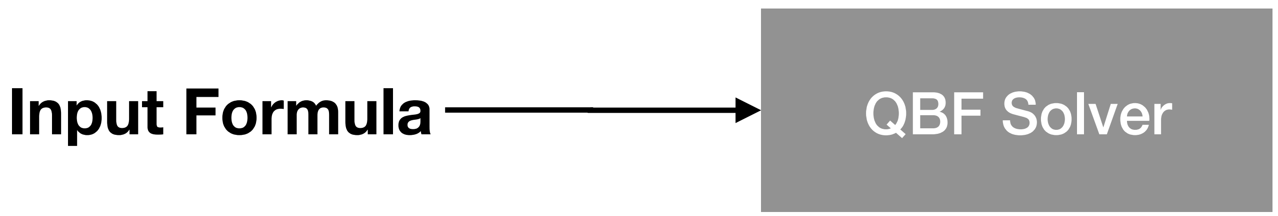
QBF Proof Complexity

QBF Proof Complexity

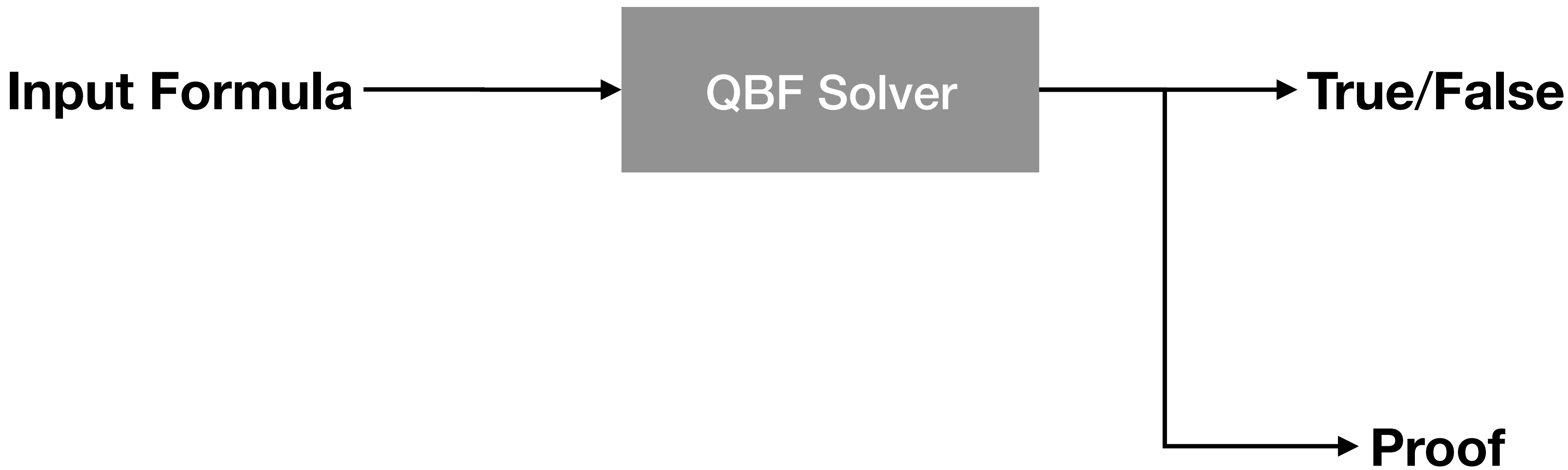
QBF Solver

QBF Proof Complexity

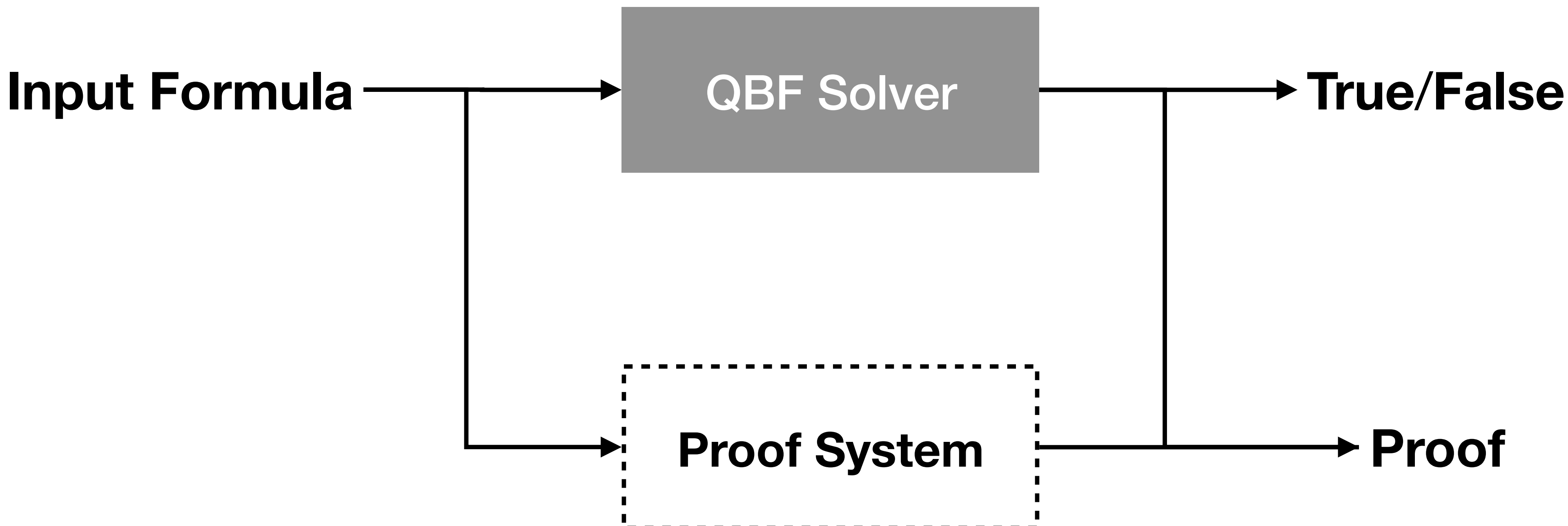


QBF Proof Complexity

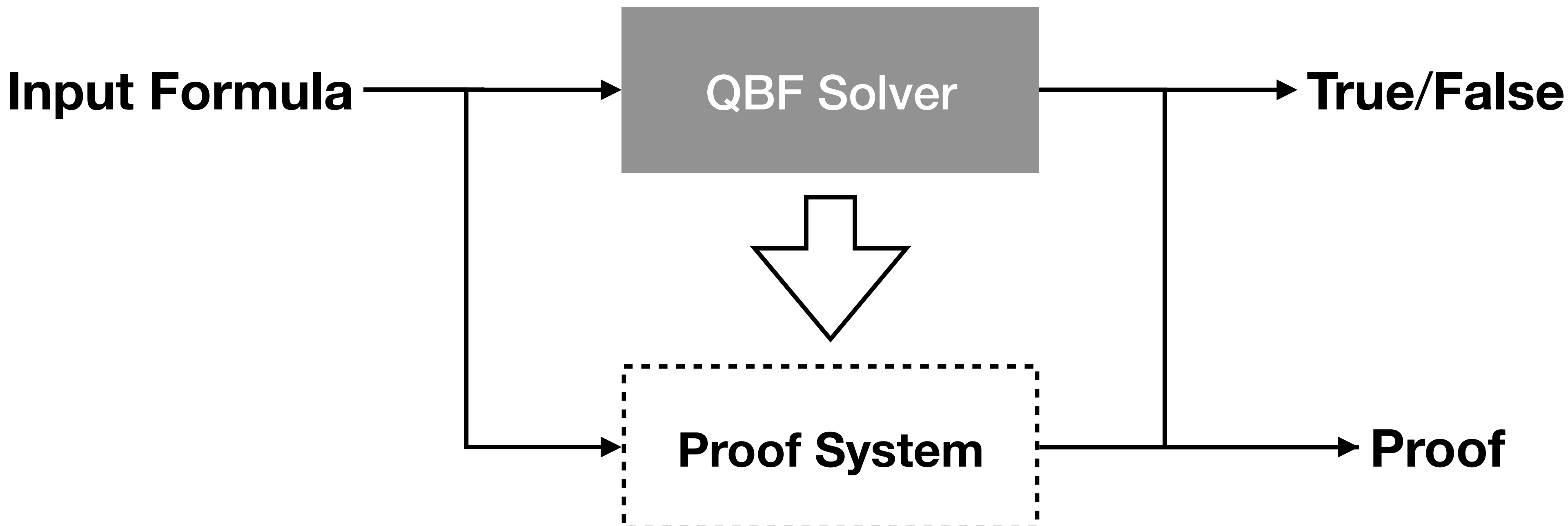
QBF Proof Complexity



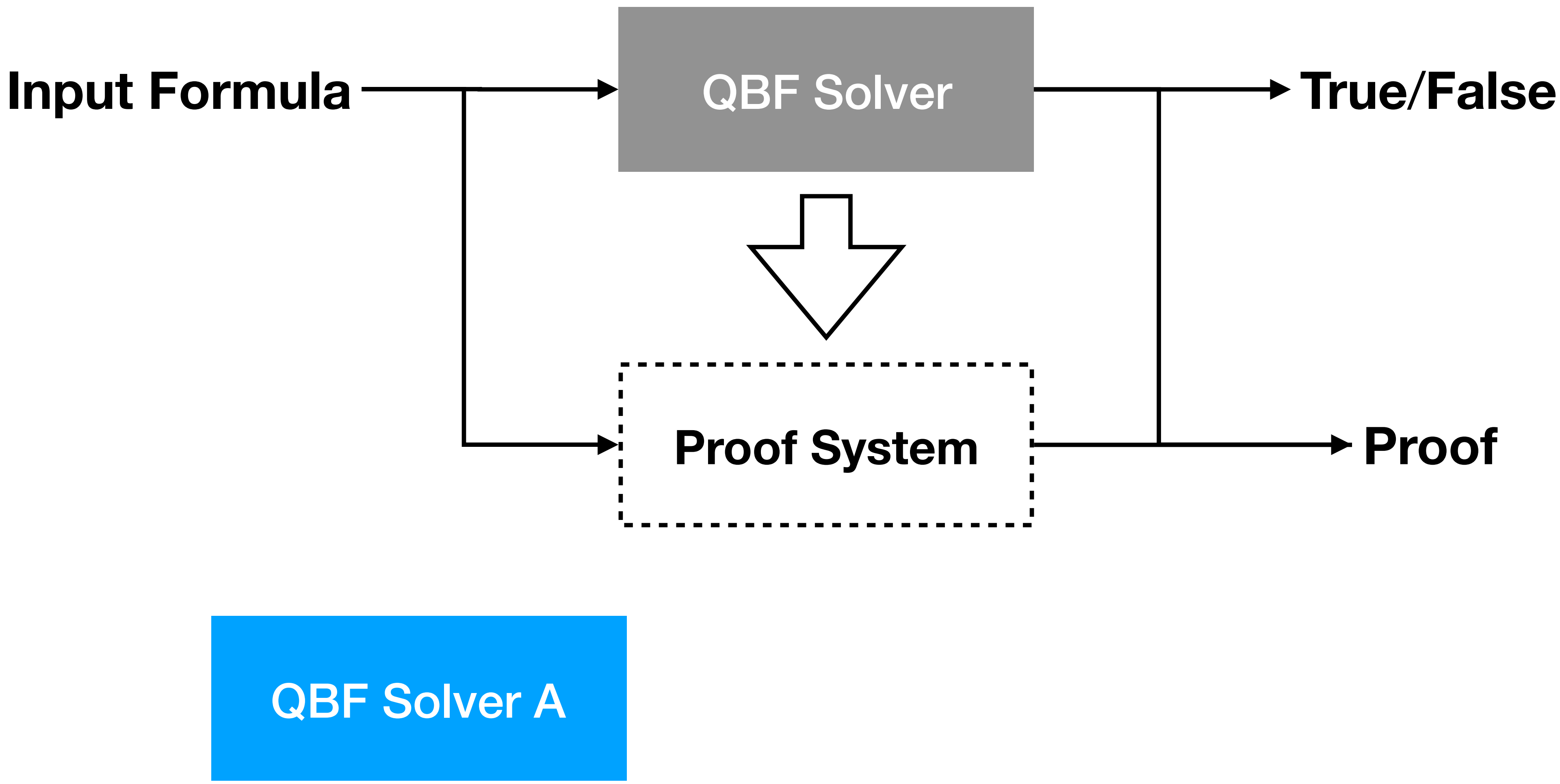
QBF Proof Complexity



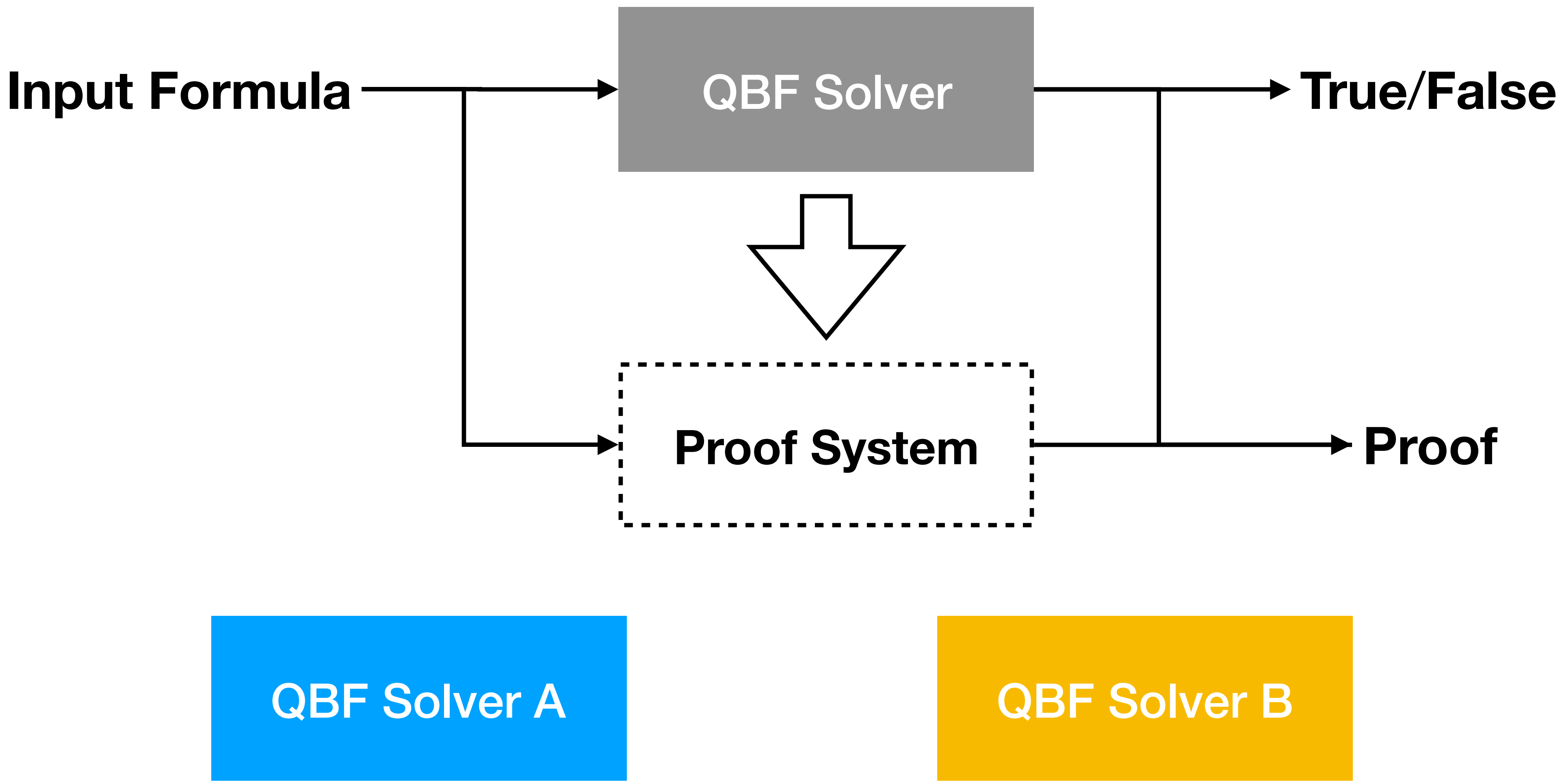
QBF Proof Complexity



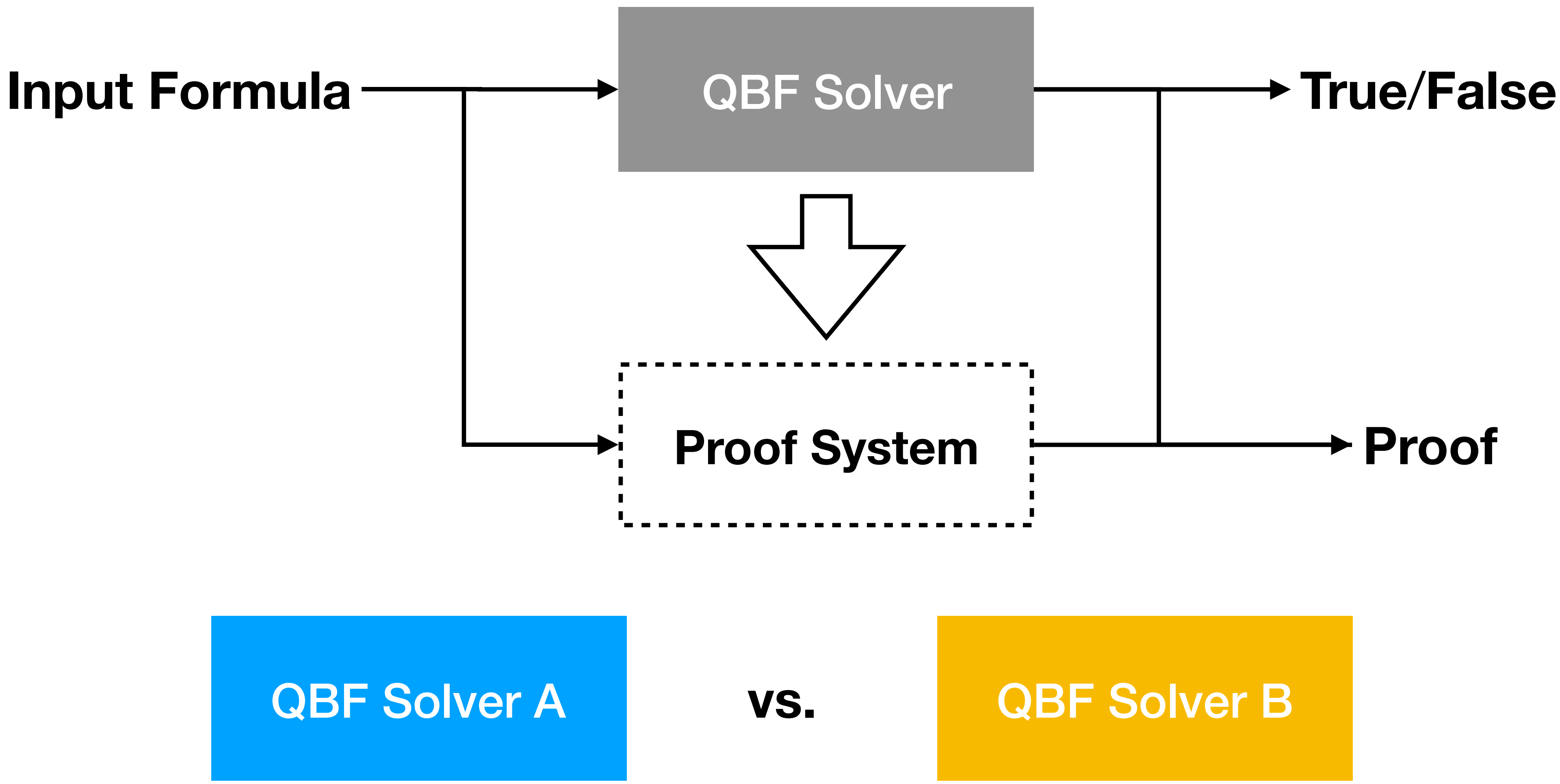
QBF Proof Complexity



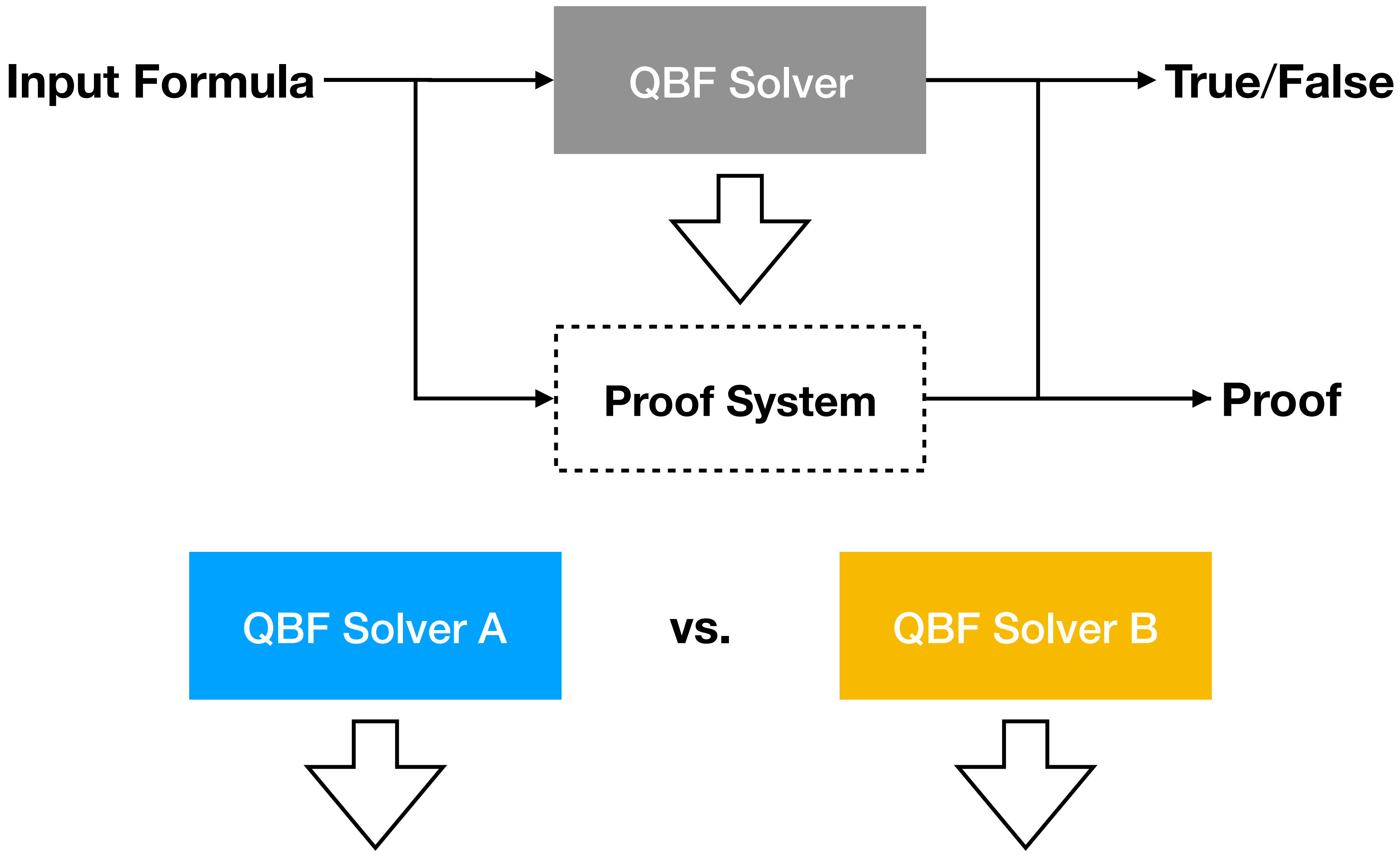
QBF Proof Complexity



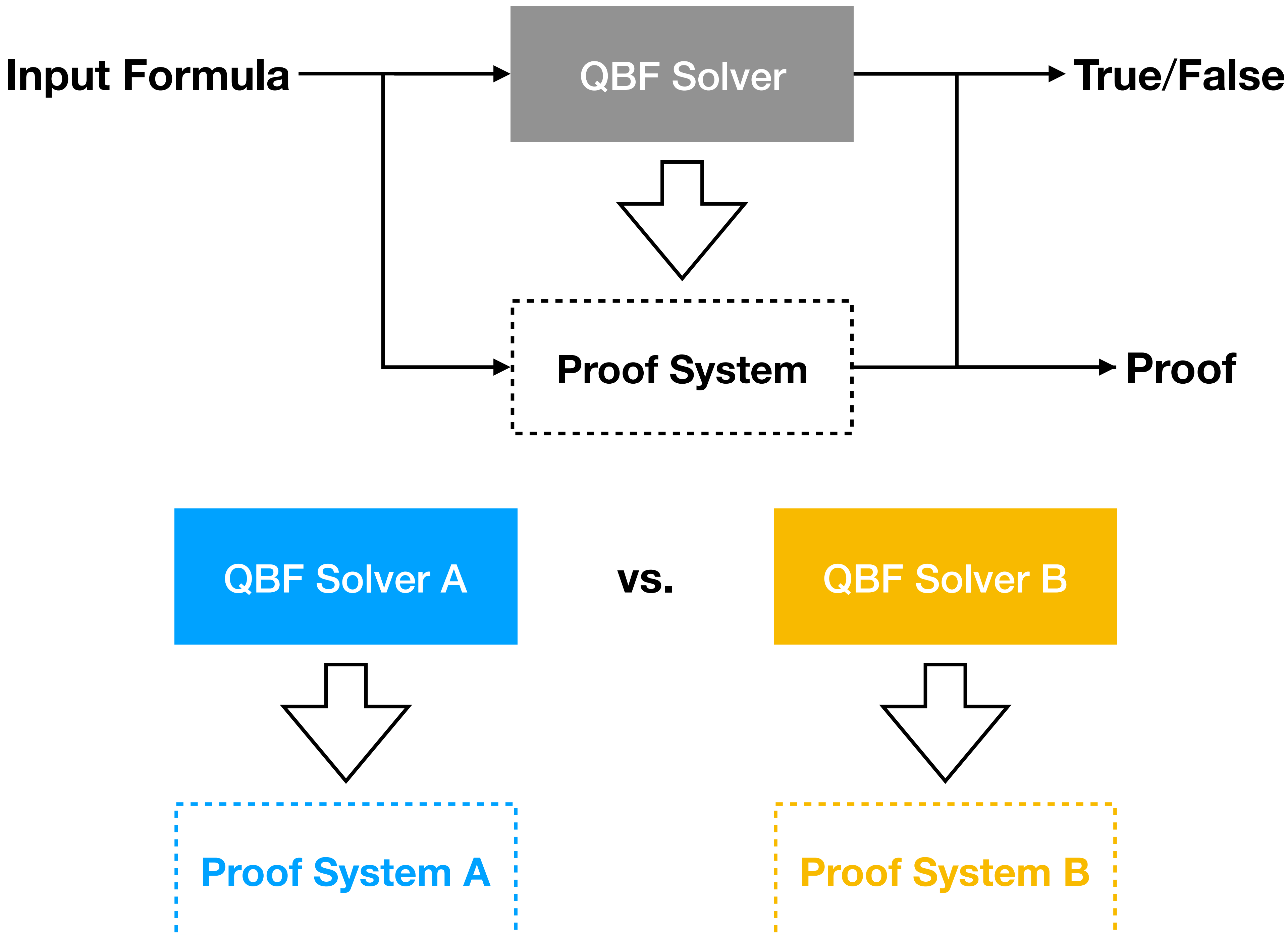
QBF Proof Complexity



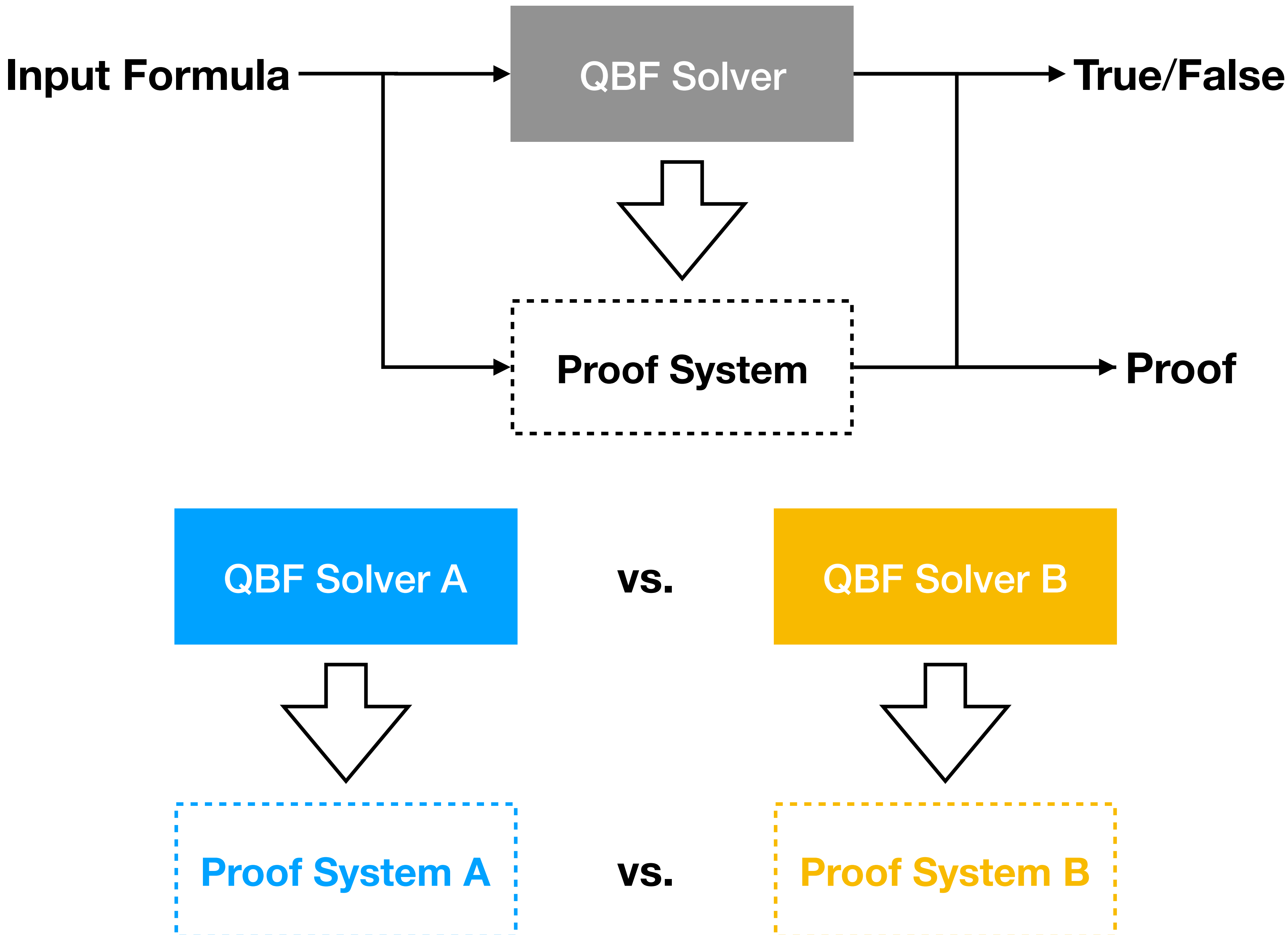
QBF Proof Complexity



QBF Proof Complexity



QBF Proof Complexity

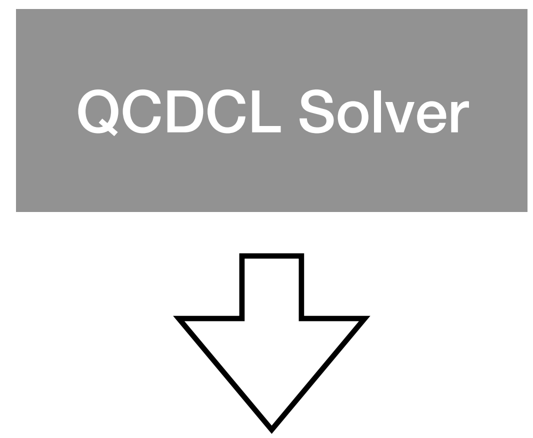


QCDCL and Q-resolution

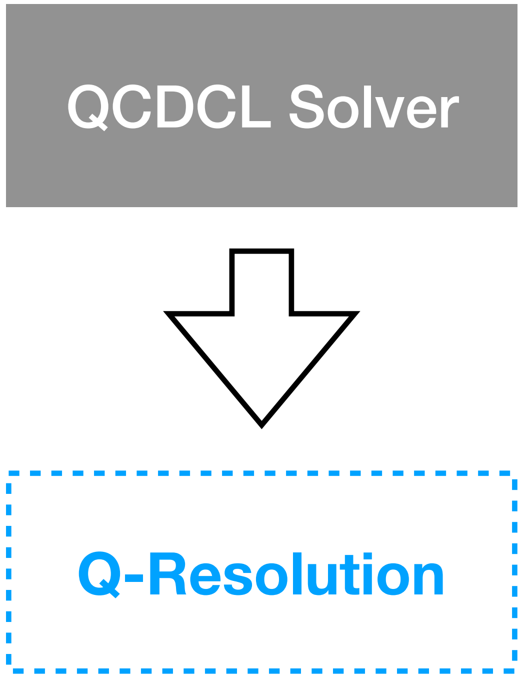
QCDCL and Q-resolution

QCDCL Solver

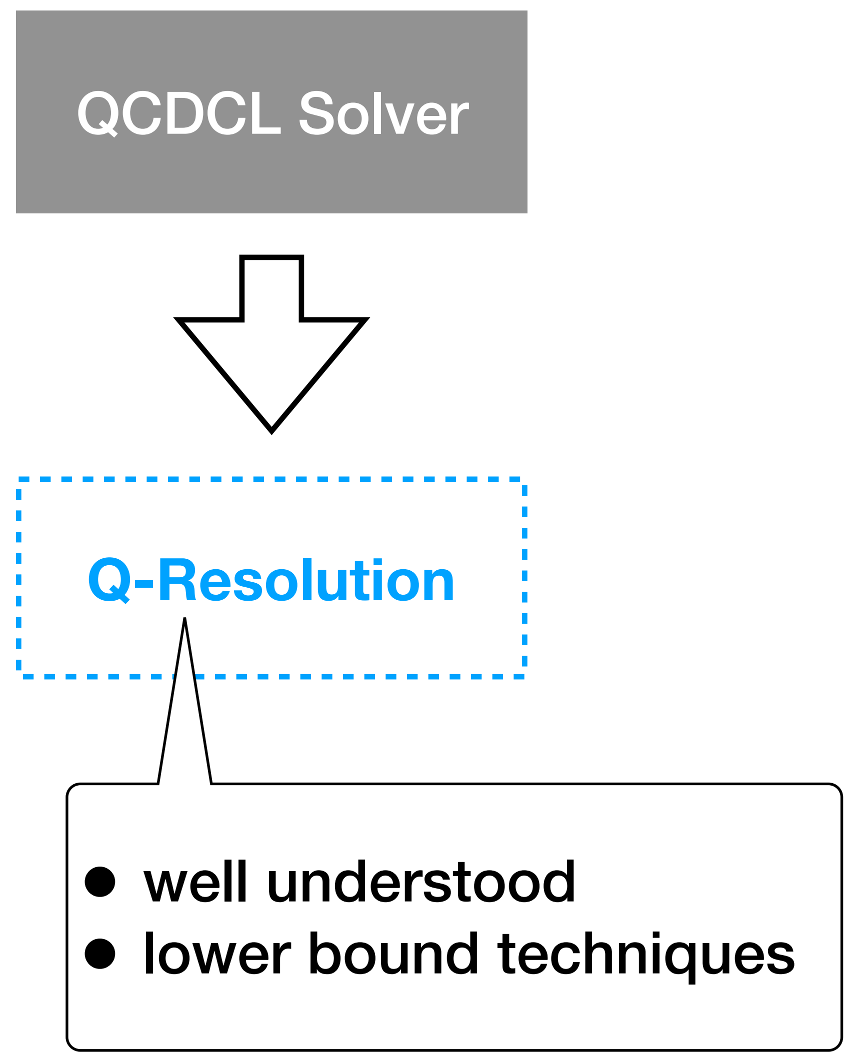
QCDCL and Q-resolution



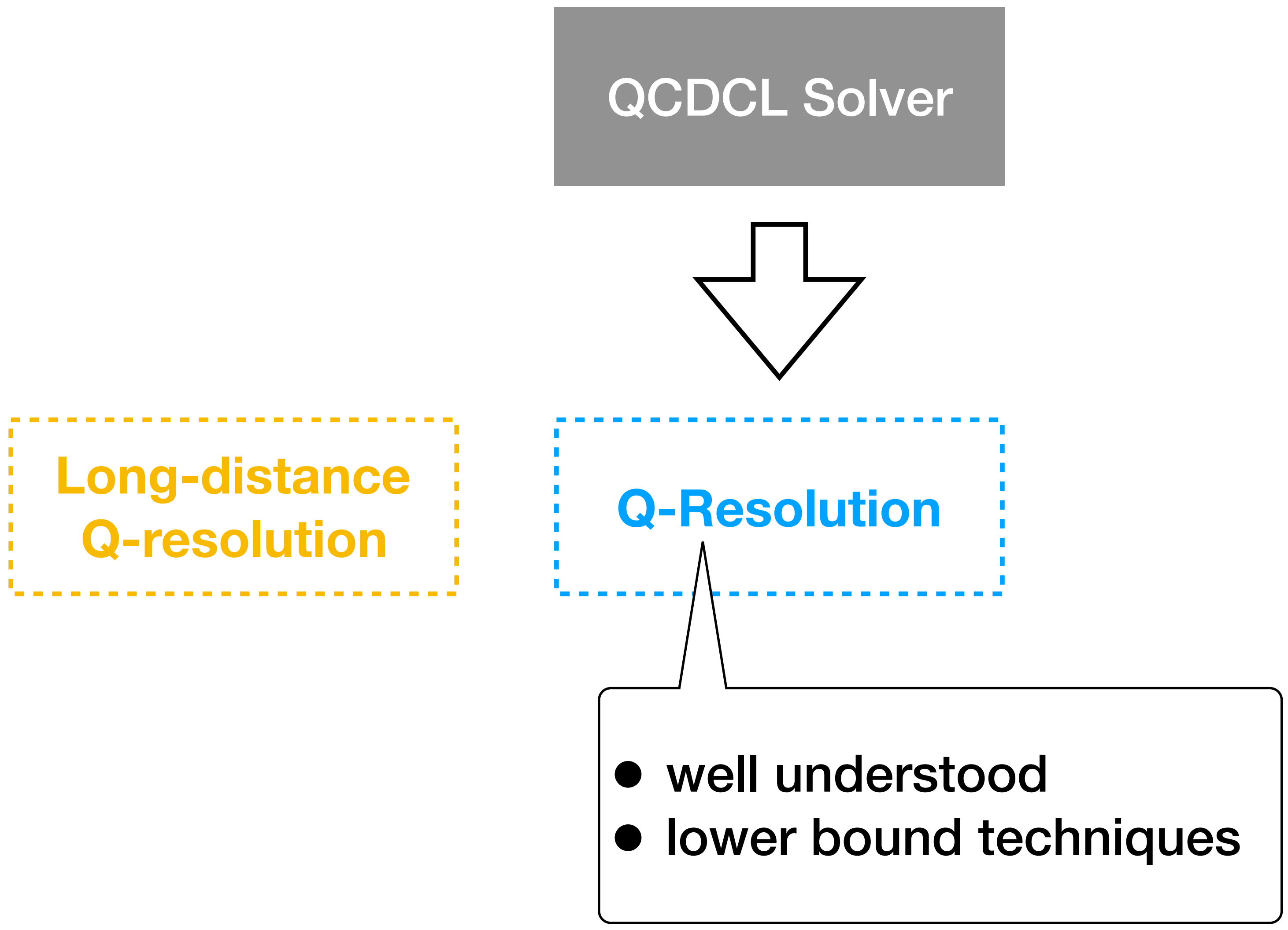
QCDCL and Q-resolution



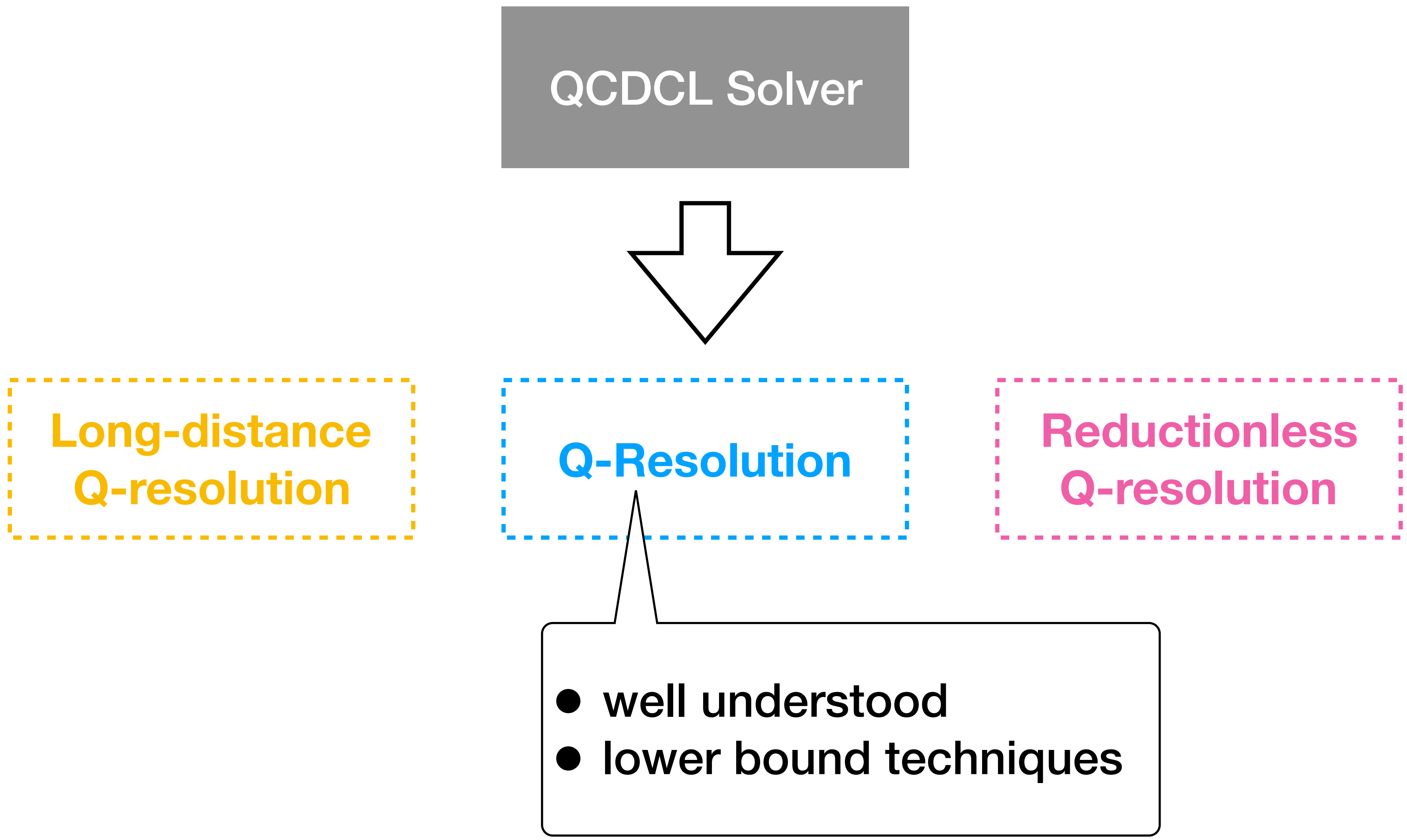
QCDCL and Q-resolution



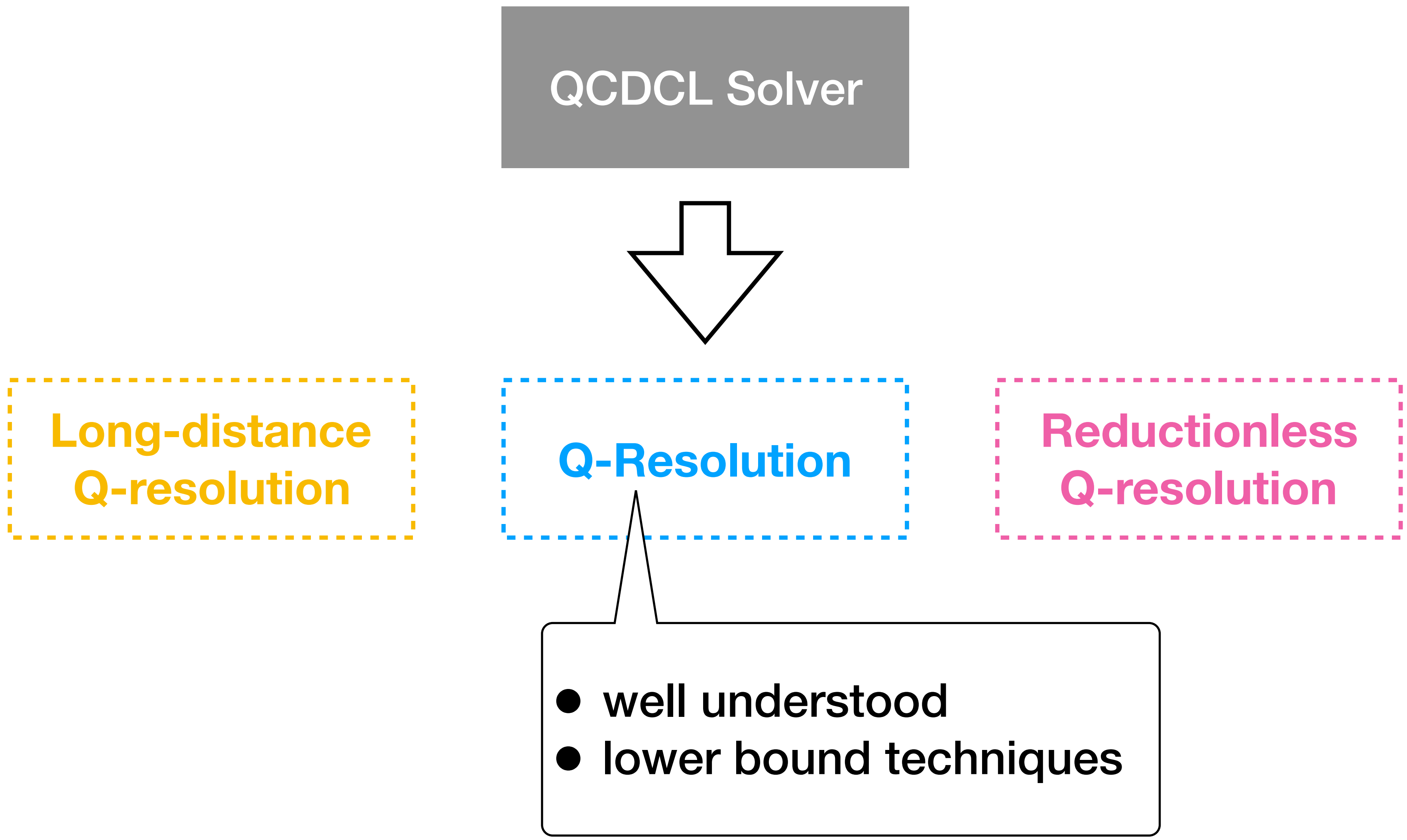
QCDCL and Q-resolution



QCDCL and Q-resolution

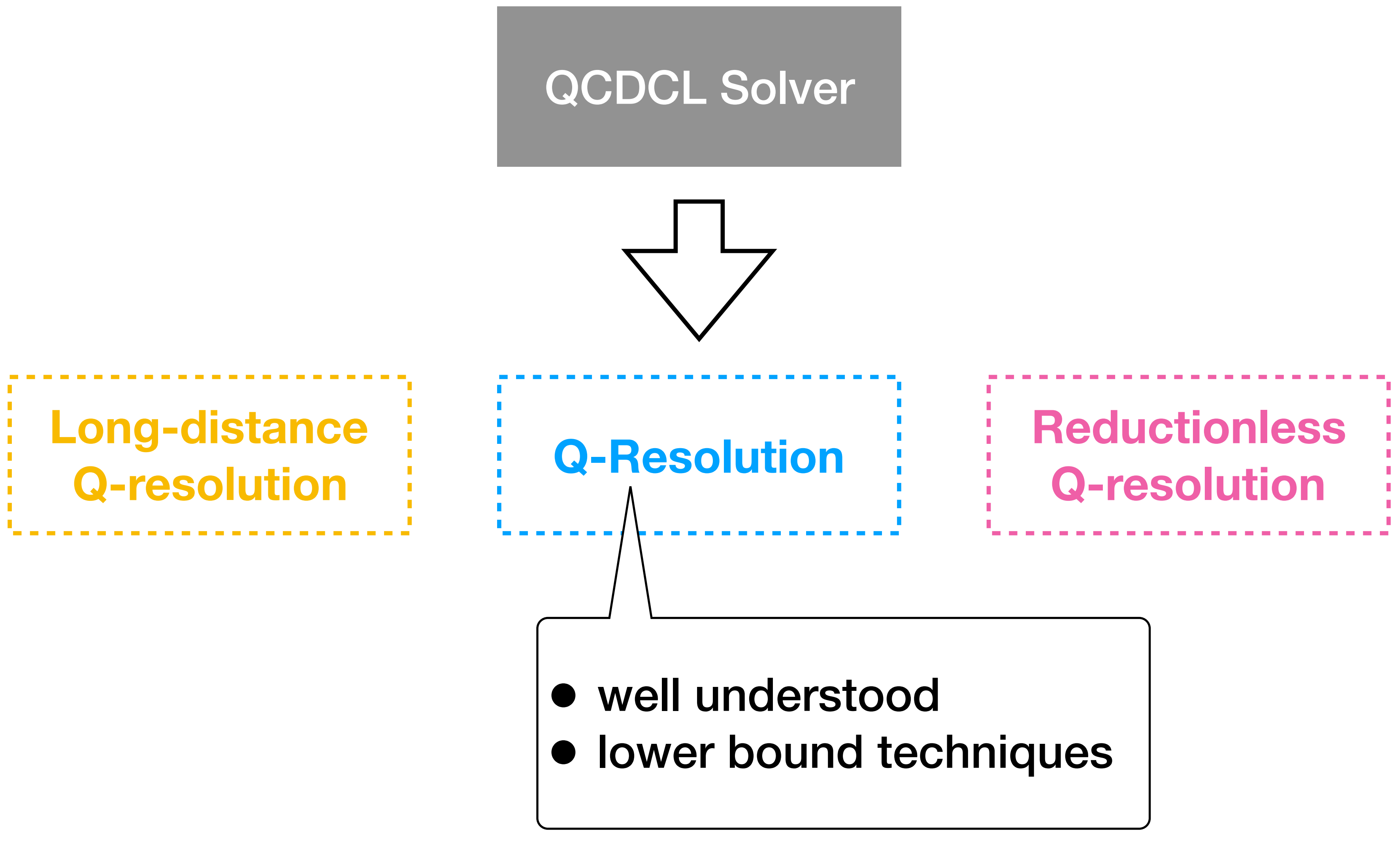


QCDCL and Q-resolution



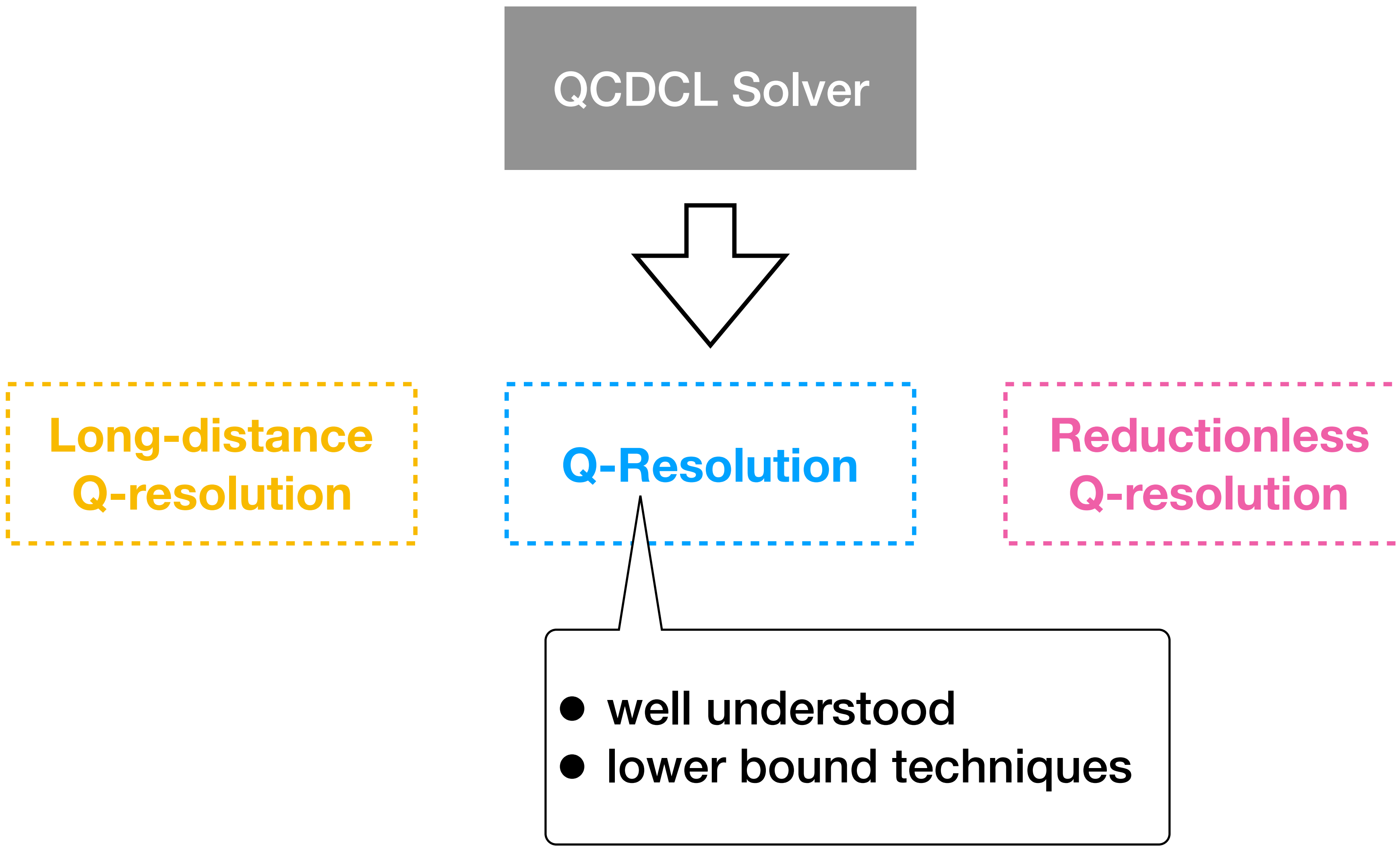
This talk:

QCDCL and Q-resolution



This talk: **1. Relative Strength Reductionless Q-res.**

QCDCL and Q-resolution



This talk:

1. **Relative Strength Reductionless Q-res.**
2. **Extending Lower Bound Techniques**

Q-resolution and Strategy Extraction

Q-resolution

Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k . \varphi$$

Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k . \varphi$$

CNF

Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k . \varphi$$ ← CNF

$$\frac{C \vee e \qquad \neg e \vee D}{C \vee D}$$

Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k \,.\, \varphi$$

CNF

$$\frac{C \vee e \qquad\qquad \neg e \vee D}{C \vee D}$$

non-tautological

Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k \,.\, \varphi$$

CNF

$$\frac{C \vee e \qquad\qquad \neg e \vee D}{C \vee D}$$

non-tautological

$$\frac{C \vee u}{C}$$

Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k \,.\, \varphi$$ ← CNF

$$\frac{C \vee e \qquad\qquad \neg e \vee D}{C \vee D}$$

↑ non-tautological

$$\frac{C \vee u}{C} \qquad e < u \text{ for } e \in C$$

Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k \,.\, \varphi$$

CNF

$$\frac{C \vee e \qquad\qquad \neg e \vee D}{C \vee D}$$

non-tautological

$$\frac{C \vee u}{C}$$

$e < u$

for $e \in C$

“universal reduction”

Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k . \varphi$$

CNF

$$\frac{C \vee e \qquad\qquad \neg e \vee D}{C \vee D}$$

non-tautological

$$\frac{C \vee u}{C} \qquad e < u \text{ for } e \in C$$

"universal reduction"

Kleine Büning, Karpinski, Flögel '95

Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k . \varphi$$

CNF

$$\frac{C \vee e \qquad \neg e \vee D}{C \vee D}$$

non-tautological

$$\frac{C \vee u}{C}$$

$e < u$
for $e \in C$

"universal reduction"

Kleine Büning, Karpinski, Flögel '95

Early work on proof complexity:

Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k \,.\, \varphi$$

CNF

$$\frac{C \vee e \qquad \neg e \vee D}{C \vee D}$$

non-tautological

$$\frac{C \vee u}{C}$$

$e < u$ for $e \in C$

"universal reduction"

Kleine Büning, Karpinski, Flögel '95

Early work on proof complexity:

1. propositional lower bounds

Q-resolution

$$\forall x_1 \exists y_1 \dots \forall x_k \exists y_k . \varphi$$

CNF

$$\frac{C \vee e \qquad \neg e \vee D}{C \vee D}$$

non-tautological

$$\frac{C \vee u}{C}$$

$e < u$
for $e \in C$

"universal reduction"

Kleine Büning, Karpinski, Flögel '95

Early work on proof complexity:

1. propositional lower bounds
2. ad-hoc proofs

Strategies

$$\forall x_1 \, \exists y_1 \, \forall x_2 \, \exists y_1 \, \ldots \forall x_n \, \exists y_1 . \, \varphi$$

Strategies

$$\forall x_1 \exists y_1 \forall x_2 \exists y_1 \ldots \forall x_n \exists y_1 . \varphi$$

Players

Strategies

$$\forall x_1 \exists y_1 \forall x_2 \exists y_1 \ldots \forall x_n \exists y_1 . \varphi$$

Players

$\forall$

Strategies

$$\forall x_1\, \exists y_1\, \forall x_2\, \exists y_1 \ldots \forall x_n\, \exists y_1 .\ \varphi$$

Players

$\forall$

$\exists$

Strategies

$$\forall x_1 \, \exists y_1 \, \forall x_2 \, \exists y_1 \, \ldots \forall x_n \, \exists y_1 . \, \varphi$$

Players

$\forall$ — T

$\exists$

Strategies

$$\forall x_1 \, \exists y_1 \, \forall x_2 \, \exists y_1 \, \ldots \forall x_n \, \exists y_1 . \, \varphi$$

Players

$\forall$	**T**	
$\exists$		**F**

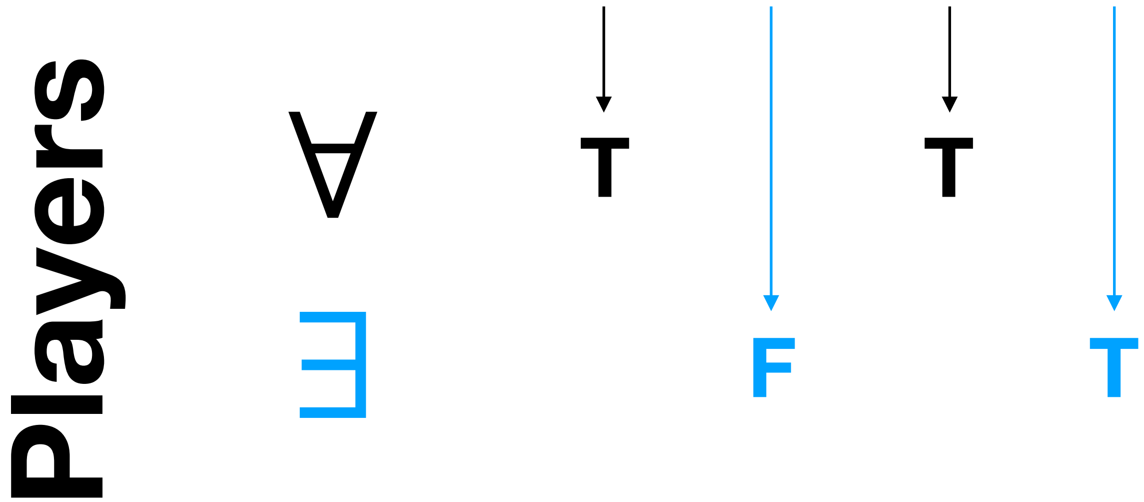
Strategies

$$\forall x_1 \exists y_1 \forall x_2 \exists y_1 \ldots \forall x_n \exists y_1 . \varphi$$

Players		x_1	y_1	x_2
	$\forall$	**T**		**T**
	$\exists$		**F**	

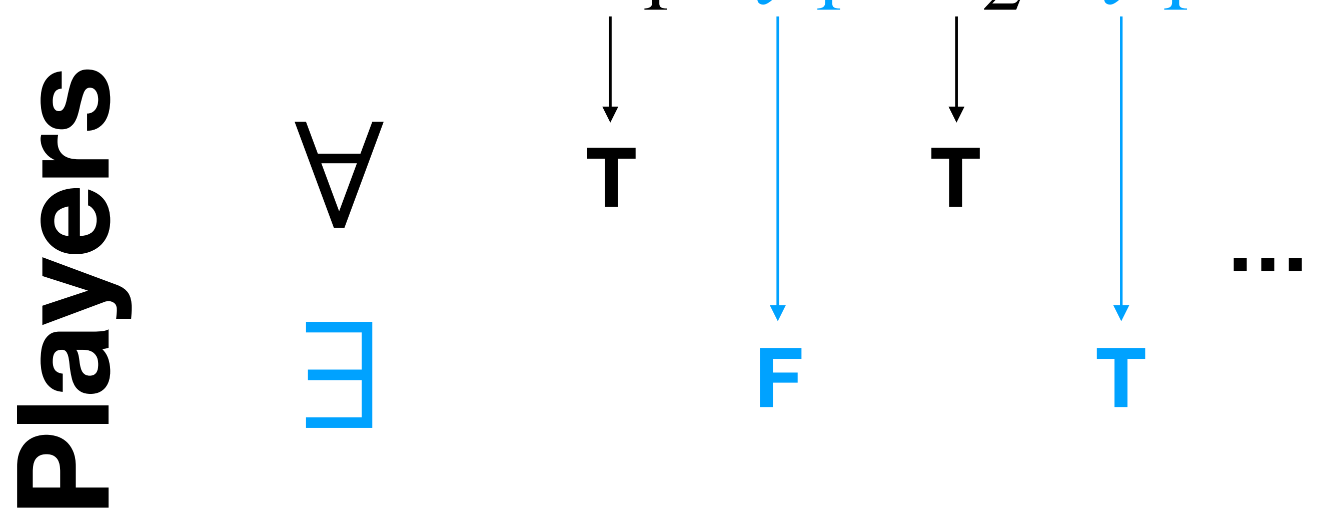
Strategies

$$\forall x_1 \, \exists y_1 \, \forall x_2 \, \exists y_1 \, \ldots \forall x_n \, \exists y_1 . \, \varphi$$



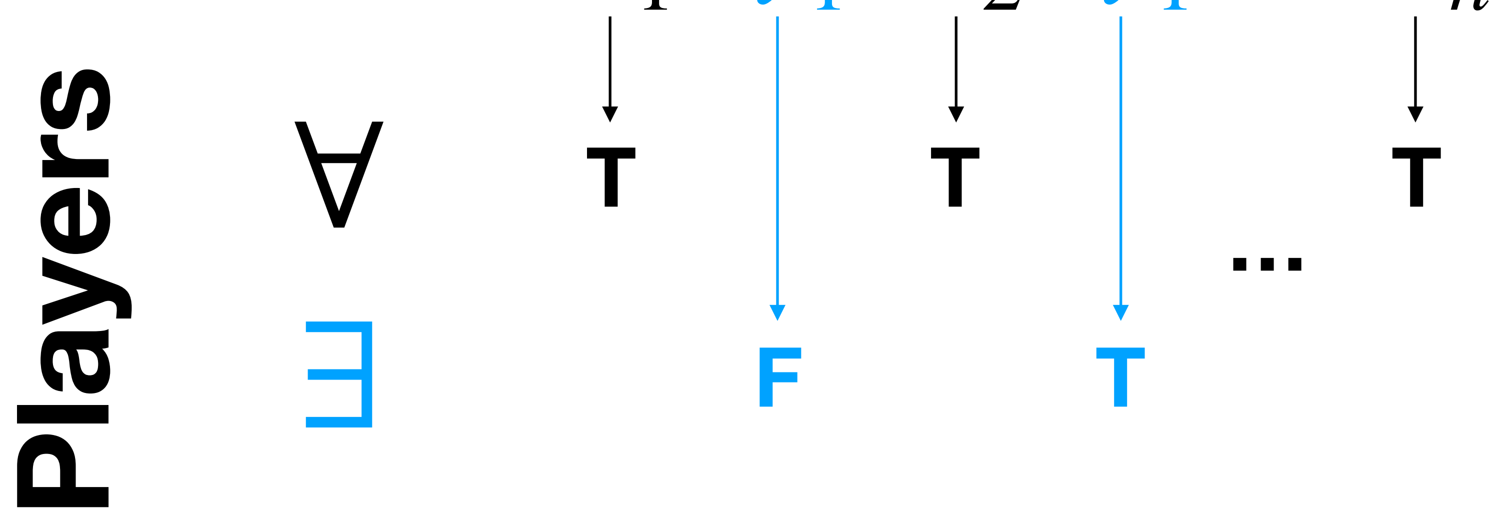
Strategies

$$\forall x_1 \, \exists y_1 \, \forall x_2 \, \exists y_1 \, \ldots \forall x_n \, \exists y_1 . \, \varphi$$

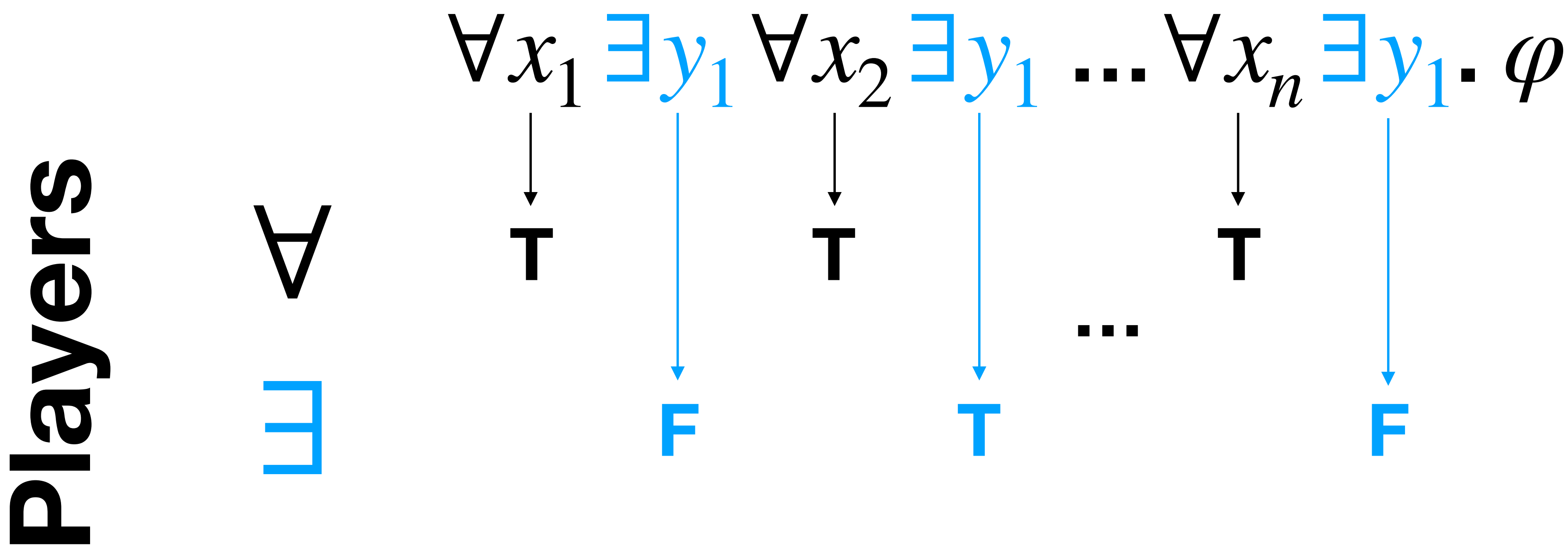


Strategies

$$\forall x_1 \, \exists y_1 \, \forall x_2 \, \exists y_1 \, \dots \forall x_n \, \exists y_1 . \, \varphi$$



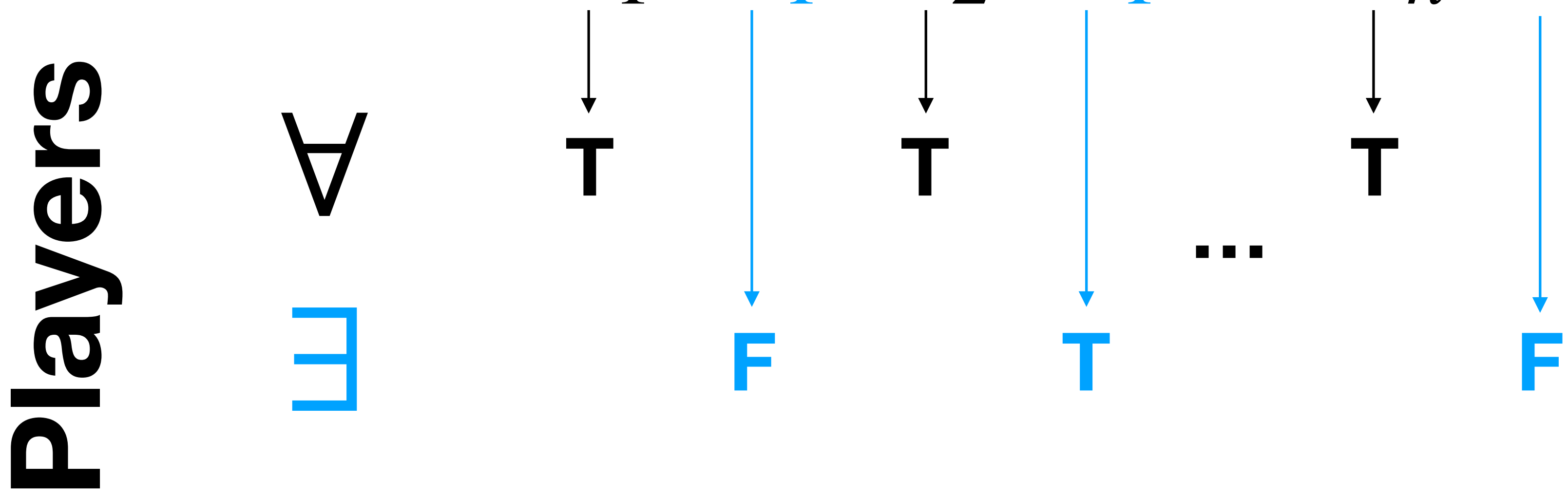
Strategies



$\forall x_1 \, \exists y_1 \, \forall x_2 \, \exists y_1 \, \ldots \, \forall x_n \, \exists y_1 . \, \varphi$
Players
$\forall$
$\exists$
T
T
T
…
F
T
F

Strategies

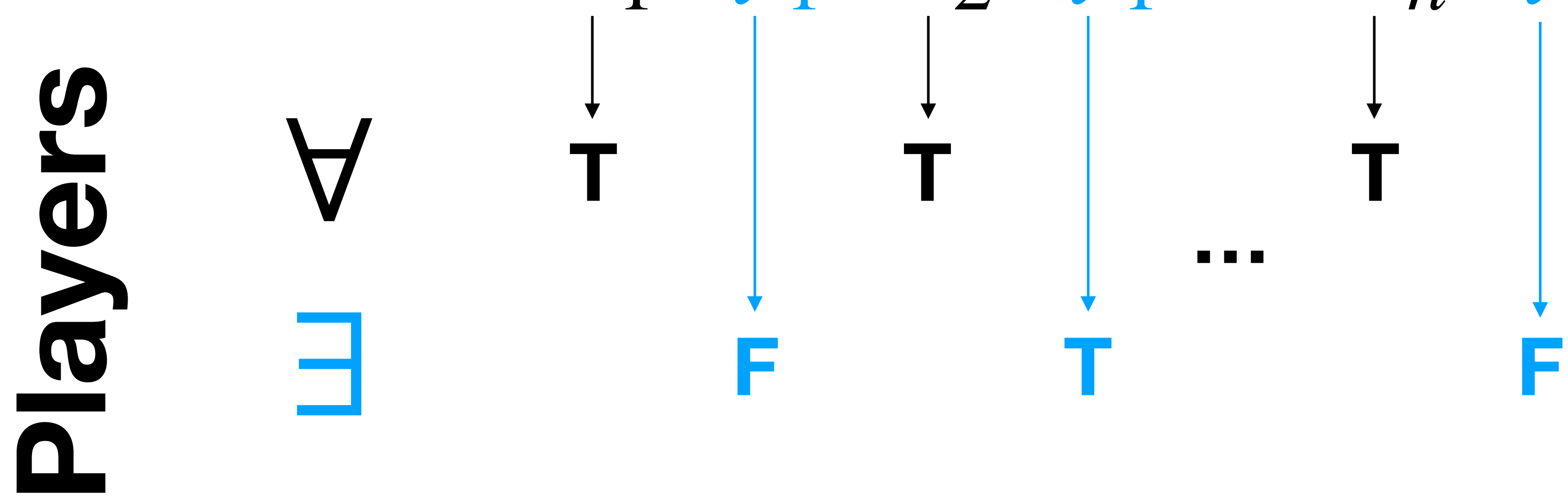
$$\forall x_1 \exists y_1 \forall x_2 \exists y_1 \ldots \forall x_n \exists y_1 . \varphi$$



∀/∃ wins if the assignment **falsifies/satisfies** φ

Strategies

$$\forall x_1 \, \exists y_1 \, \forall x_2 \, \exists y_1 \ldots \forall x_n \, \exists y_1 . \, \varphi$$



∀/∃ wins if the assignment **falsifies/satisfies** φ

true ⇔ existential winning strategy

Strategies

$$\forall x_1 \, \exists y_1 \, \forall x_2 \, \exists y_1 \ldots \forall x_n \, \exists y_1 . \, \varphi$$

Players							
$\forall$	T		T		...	T	
$\exists$		F		T			F

$\forall$/$\exists$ wins if the assignment **falsifies/satisfies** φ

true ⇔ existential winning strategy

false ⇔ universal winning strategy

Strategy Extraction
for Q-resolution

Strategy Extraction for Q-resolution

Proof

Strategy Extraction for Q-resolution

Proof

C_1

Strategy Extraction for Q-resolution

Proof

C_1

C_2

Strategy Extraction for Q-resolution

Proof

C_1

C_2

C_3

Strategy Extraction for Q-resolution

Proof

C_1

C_2

C_3

...

Strategy Extraction for Q-resolution

Proof

C_1

C_2

C_3

...

C_{k-1}

Strategy Extraction for Q-resolution

Proof

C_1

C_2

C_3

...

C_{k-1}

$\square$

Strategy Extraction for Q-resolution

Proof

C_1

C_2

C_3

...

C_{k-1}

$\square$

Strategy Extraction for Q-resolution

Proof → **Strategy**

C_1

C_2

C_3

...

C_{k-1}

$\square$

Strategy Extraction for Q-resolution

Proof

C_1

C_2

C_3

...

C_{k-1}

$\square$

→

Strategy

$f_{u_1}, \ldots, f_{u_n}$

Strategy Extraction for Q-resolution

Balabanov & Jiang '12

Proof

C_1

C_2

C_3

...

C_{k-1}

$\square$

Strategy

$f_{u_1}, \ldots, f_{u_n}$

Strategy Extraction for Q-resolution

Balabanov & Jiang '12

Proof ⇒ **Strategy**

C_1

C_2

C_3

...

C_{k-1}

$\square$

$$\frac{C_3 \vee u}{C_3}$$

$f_{u_1}, \ldots, f_{u_n}$

Strategy Extraction for Q-resolution

Balabanov & Jiang '12

Proof

C_1

C_2

C_3

...

C_{k-1}

$\Box$

$$\frac{C_3 \vee u}{C_3}$$

Strategy

$f_{u_1}, \ldots, f_{u_n}$

if $\neg C_3$ then $\neg u$

Strategy Extraction for Q-resolution

Balabanov & Jiang '12

Proof ⇒ **Strategy**

C_1

C_2

C_3

$\dfrac{C_3 \vee u}{C_3}$

...

C_{k-1}

$\dfrac{C_{k-1} \vee \neg u}{C_{k-1}}$

$\square$

$f_{u_1}, \ldots, f_{u_n}$

if $\neg C_3$ then $\neg u$

Strategy Extraction for Q-resolution

Balabanov & Jiang '12

Proof ⇒ **Strategy**

C_1

C_2

C_3

...

C_{k-1}

$\square$

$$\frac{C_3 \vee u}{C_3}$$

$$\frac{C_{k-1} \vee \neg u}{C_{k-1}}$$

$f_{u_1}, \ldots, f_{u_n}$

if $\neg C_3$ then $\neg u$

else if $\neg C_{k-1}$ then u

Strategy Extraction for Q-resolution

Balabanov & Jiang '12

Proof

C_1

C_2

C_3

...

C_{k-1}

$\square$

$$\frac{C_3 \vee u}{C_3}$$

$$\frac{C_{k-1} \vee \neg u}{C_{k-1}}$$

Strategy

$f_{u_1}, \ldots, f_{u_n}$

if $\neg C_3$ then $\neg u$

else if $\neg C_{k-1}$ then u

...

Strategy Extraction for Q-resolution

Balabanov & Jiang '12

Proof ⇒ **Strategy**

C_1

C_2

C_3

...

C_{k-1}

$\square$

$$\frac{C_3 \vee u}{C_3}$$

$$\frac{C_{k-1} \vee \neg u}{C_{k-1}}$$

$f_{u_1}, \ldots, f_{u_n}$

f_u:

if $\neg C_3$ then $\neg u$

else if $\neg C_{k-1}$ then u

...

Proof Size Lower Bounds via Strategy Extraction

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u

else if $\neg C_2$ then l_2^u

...

else if $\neg C_k$ then l_k^u

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u

else if $\neg C_2$ then l_2^u ← $l_i^u \in \{u, \neg u\}$

...

else if $\neg C_k$ then l_k^u

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u

else if $\neg C_2$ then l_2^u

...

else if $\neg C_k$ then l_k^u

$l_i^u \in \{u, \neg u\}$

$\neg C_1 \rightarrow l_1^u$

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u

else if $\neg C_2$ then l_2^u

...

else if $\neg C_k$ then l_k^u

$l_i^u \in \{u, \neg u\}$

$\neg C_1 \rightarrow l_1^u \;\; \wedge$

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u

else if $\neg C_2$ then l_2^u ← $l_i^u \in \{u, \neg u\}$

...

else if $\neg C_k$ then l_k^u

$\neg C_1 \rightarrow l_1^u \quad \wedge$

$(C_1 \wedge \neg C_2) \rightarrow l_2^u$

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u

else if $\neg C_2$ then l_2^u ← $l_i^u \in \{u, \neg u\}$

...

else if $\neg C_k$ then l_k^u

$$\neg C_1 \rightarrow l_1^u \quad \wedge$$
$$(C_1 \wedge \neg C_2) \rightarrow l_2^u \quad \wedge$$

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u

else if $\neg C_2$ then l_2^u ← $l_i^u \in \{u, \neg u\}$

...

else if $\neg C_k$ then l_k^u

$\neg C_1 \rightarrow l_1^u \;\; \wedge$

$(C_1 \wedge \neg C_2) \rightarrow l_2^u \;\; \wedge$

...

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u

else if $\neg C_2$ then l_2^u

...

else if $\neg C_k$ then l_k^u

$l_i^u \in \{u, \neg u\}$

$$\neg C_1 \rightarrow l_1^u \quad \wedge$$

$$(C_1 \wedge \neg C_2) \rightarrow l_2^u \quad \wedge$$

...

$$(\bigwedge_{i=1}^{k-1} C_i \wedge \neg C_k) \rightarrow l_k^u$$

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u

else if $\neg C_2$ then l_2^u ← $l_i^u \in \{u, \neg u\}$

...

else if $\neg C_k$ then l_k^u

$$\neg C_1 \rightarrow l_1^u \quad \wedge$$

$$(C_1 \wedge \neg C_2) \rightarrow l_2^u \quad \wedge$$

...

$$(\bigwedge_{i=1}^{k-1} C_i \wedge \neg C_k) \rightarrow l_k^u$$

$$\exists y_1 \ldots \exists y_n \forall u \,.\, (u \nleftrightarrow f(y_1, \ldots, y_n))$$

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u
else if $\neg C_2$ then l_2^u
...
else if $\neg C_k$ then l_k^u

$l_i^u \in \{u, \neg u\}$

$\neg C_1 \rightarrow l_1^u \quad \wedge$

$(C_1 \wedge \neg C_2) \rightarrow l_2^u \quad \wedge$

...

$(\bigwedge_{i=1}^{k-1} C_i \wedge \neg C_k) \rightarrow l_k^u$

$$\exists y_1 \ldots \exists y_n \forall u \,.\, (u \nleftrightarrow f(y_1, \ldots, y_n))$$

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u

else if $\neg C_2$ then l_2^u

...

else if $\neg C_k$ then l_k^u

$l_i^u \in \{u, \neg u\}$

$$\neg C_1 \rightarrow l_1^u \;\wedge$$

$$(C_1 \wedge \neg C_2) \rightarrow l_2^u \;\wedge$$

...

$$(\bigwedge_{i=1}^{k-1} C_i \wedge \neg C_k) \rightarrow l_k^u$$

$$\in AC^0$$

$$\exists y_1 \ldots \exists y_n \forall u \,.\, (u \nleftrightarrow f(y_1, \ldots, y_n))$$

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u
else if $\neg C_2$ then l_2^u ← $l_i^u \in \{u, \neg u\}$
...
else if $\neg C_k$ then l_k^u

$\neg C_1 \rightarrow l_1^u \quad \wedge$

$(C_1 \wedge \neg C_2) \rightarrow l_2^u \quad \wedge$

...

$(\bigwedge_{i=1}^{k-1} C_i \wedge \neg C_k) \rightarrow l_k^u$

$\in AC^0$

$$\exists y_1 \ldots \exists y_n \forall u \, . \, (u \nleftrightarrow f(y_1, \ldots, y_n))$$

$$\exists y_1 \ldots \exists y_n \forall u \, . \, (u \nleftrightarrow \oplus (y_1, \ldots, y_n))$$

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u
else if $\neg C_2$ then l_2^u
...
else if $\neg C_k$ then l_k^u

$l_i^u \in \{u, \neg u\}$

$$\neg C_1 \to l_1^u \;\wedge$$
$$(C_1 \wedge \neg C_2) \to l_2^u \;\wedge$$
...
$$(\bigwedge_{i=1}^{k-1} C_i \wedge \neg C_k) \to l_k^u$$

$\in AC^0$

$$\exists y_1 \ldots \exists y_n \forall u\,.\,(u \not\leftrightarrow f(y_1, \ldots, y_n))$$

$$\exists y_1 \ldots \exists y_n \forall u\,.\,(u \not\leftrightarrow \oplus\,(y_1, \ldots, y_n))$$ **QParity**

Proof Size Lower Bounds via Strategy Extraction

Beyersdorff, Chew, Janota '15

if $\neg C_1$ then l_1^u
else if $\neg C_2$ then l_2^u
...
else if $\neg C_k$ then l_k^u

$l_i^u \in \{u, \neg u\}$

$$\neg C_1 \rightarrow l_1^u \;\wedge$$
$$(C_1 \wedge \neg C_2) \rightarrow l_2^u \;\wedge$$
$$\ldots$$
$$(\bigwedge_{i=1}^{k-1} C_i \wedge \neg C_k) \rightarrow l_k^u$$

$$\in AC^0$$

$$\exists y_1 \ldots \exists y_n \forall u\,.\,(u \not\leftrightarrow f(y_1, \ldots, y_n))$$

$$\exists y_1 \ldots \exists y_n \forall u\,.\,(u \not\leftrightarrow \oplus\,(y_1, \ldots, y_n))$$ **QParity**

Theorem

QParity does not have polynomial Q-resolution proofs.

Long-Distance Q-resolution

Long-Distance Q-resolution

Long-Distance Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k . \varphi$$

Long-Distance Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k \,.\, \varphi$$

$e \vee u \vee C$

Long-Distance Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k . \varphi$$

$e \vee u \vee C \qquad \neg e \vee \neg u \vee D$

Long-Distance Q-resolution

$$\forall x_1 \exists y_1 \dots \forall x_k \exists y_k \,.\, \varphi$$

$$\frac{e \vee u \vee C \qquad \neg e \vee \neg u \vee D}{}$$

Long-Distance Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k \, . \, \varphi$$

$$\frac{e \vee u \vee C \qquad \neg e \vee \neg u \vee D}{C \vee D \vee u^*}$$

Long-Distance Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k . \varphi$$

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"merged literal"

Long-Distance Q-resolution

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“merged literal”

$$\frac{u \qquad u^*}{u^*}$$

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$$\forall x_1 \exists y_1 \dots \forall x_k \exists y_k \,.\, \varphi$$

$$\frac{e \vee u \vee C \qquad \neg e \vee \neg u \vee D}{C \vee D \vee u^*} \qquad \frac{C \vee u^*}{C}$$

“merged literal”

$$\frac{u \qquad u^*}{u^*} \qquad \frac{\neg u \qquad u^*}{u^*} \qquad \frac{u^* \qquad u^*}{u^*}$$

LDQ-resolution emerges from conflict analysis in QCDCL.

Long-Distance Q-resolution

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Reductionless Q-resolution

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Reductionless Q-resolution can be used in (prefix-ordered) QCDCL.

Reductionless Q-resolution

$$\forall x_1 \exists y_1 \ldots \forall x_k \exists y_k \,.\, \varphi$$

$$\frac{e \vee u \vee C \qquad \neg e \vee \neg u \vee D}{C \vee D \vee u^*}$$

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$$\frac{C \vee u^*}{C}$$

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Reductionless Q-resolution can be used in (prefix-ordered) QCDCL.

Bjørner, Janota, Klieber '15

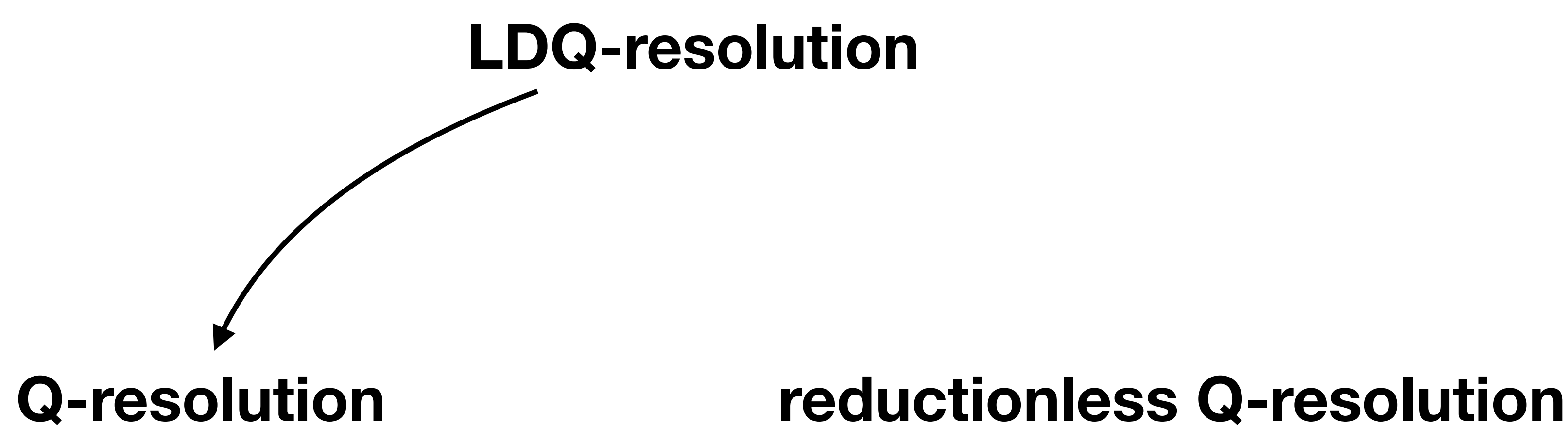
Proof Complexity

LDQ-resolution

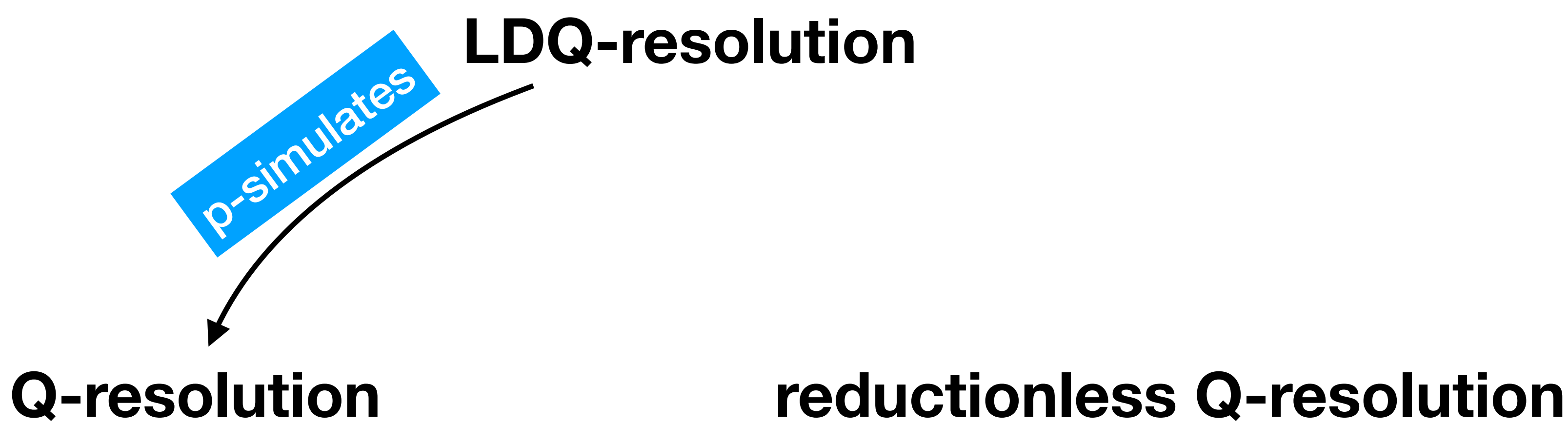
Q-resolution

reductionless Q-resolution

Proof Complexity



Proof Complexity



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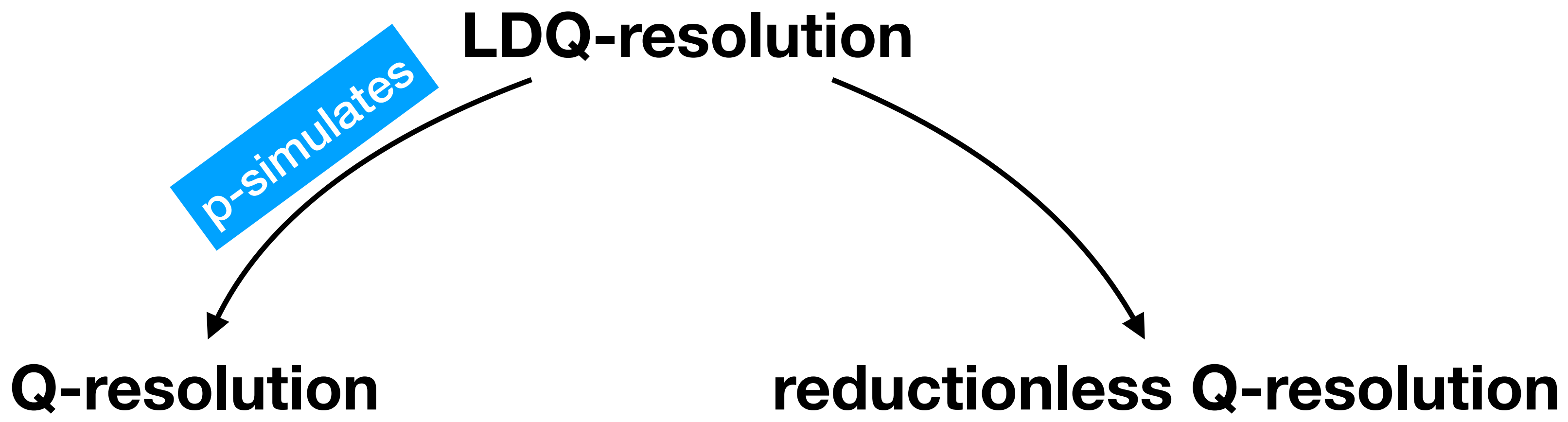

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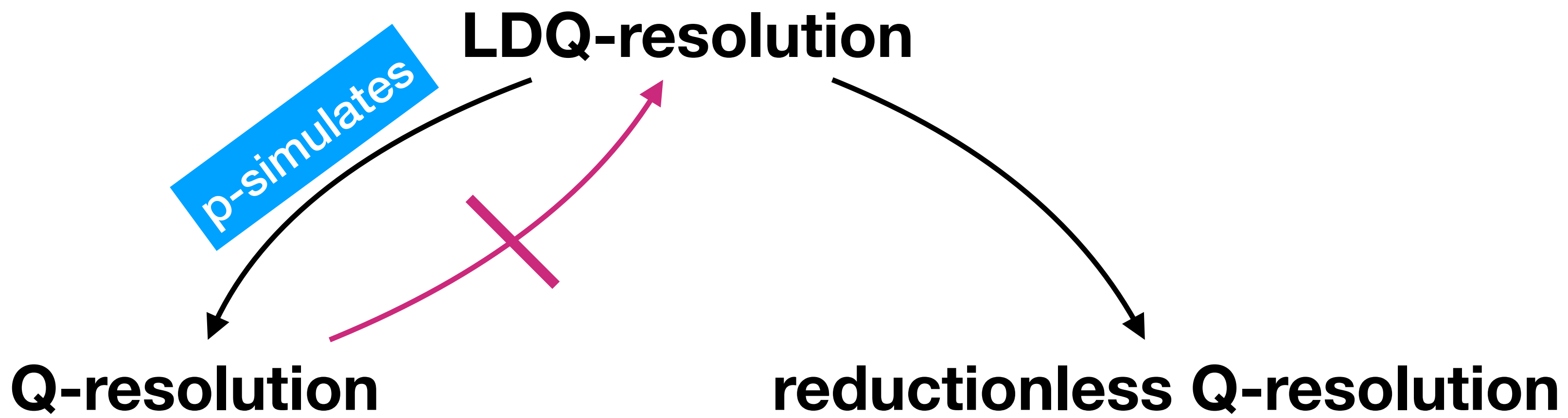

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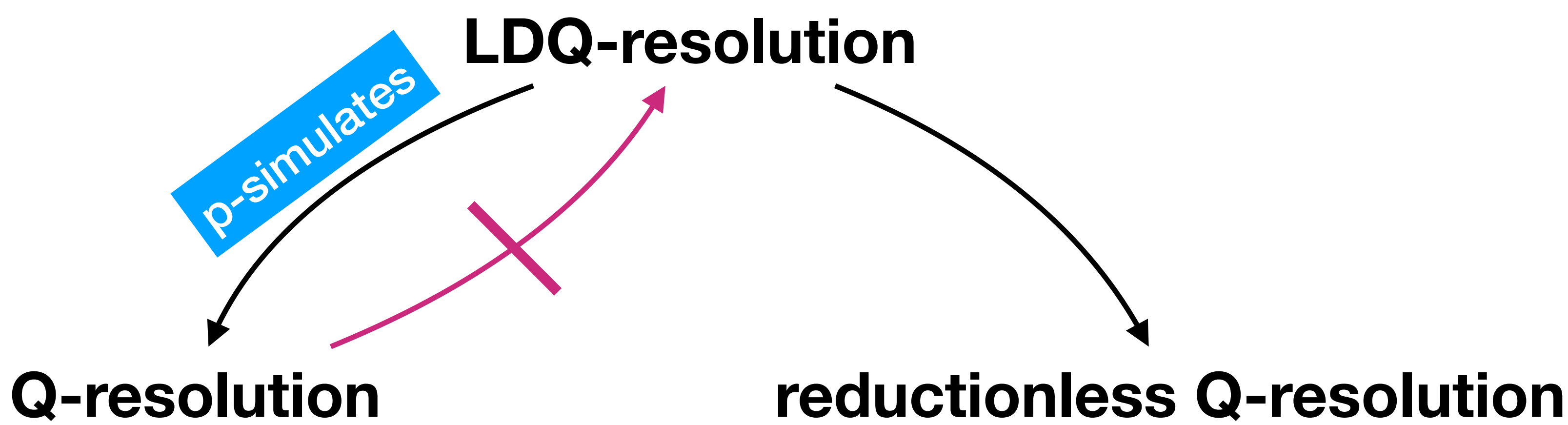
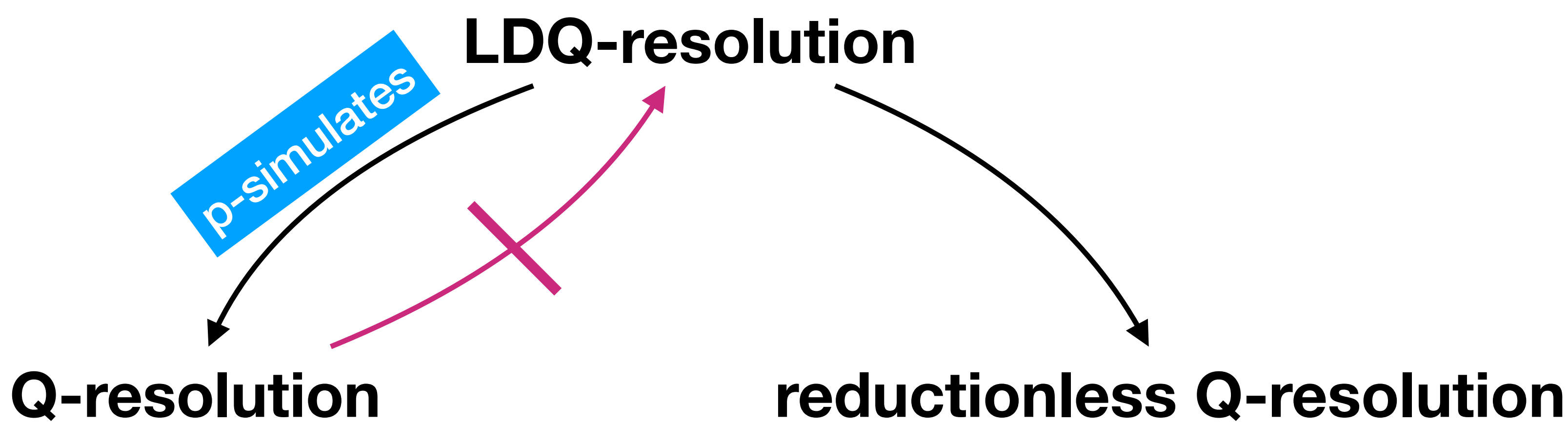


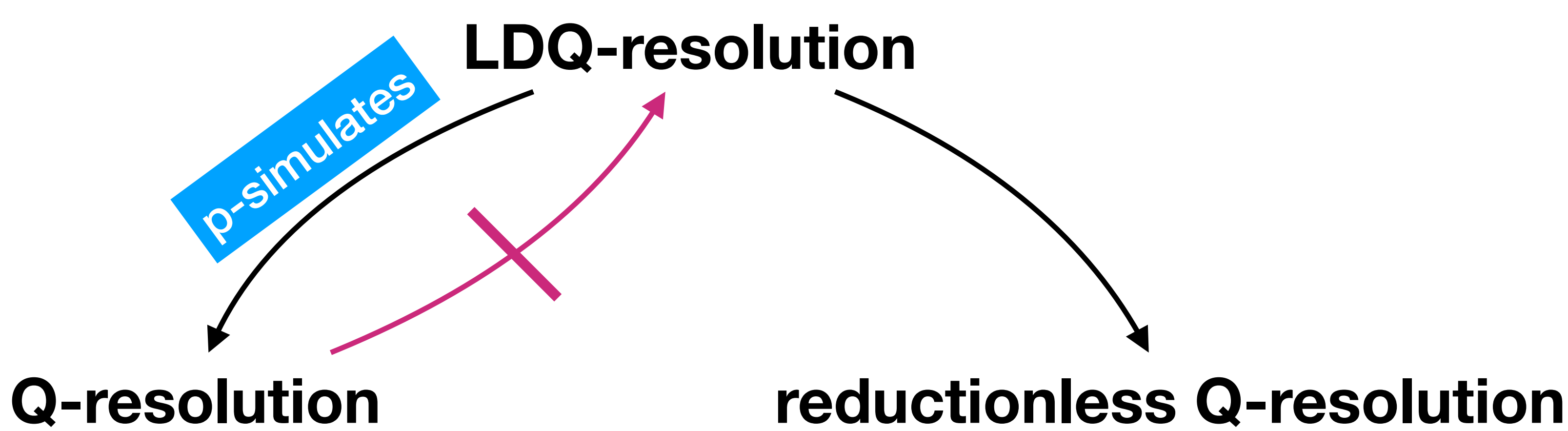
QParity does not have polynomial Q-resolution proofs.

Proof Complexity



QParity does not have polynomial Q-resolution proofs. Beyersdorff, Chew, Janota'15

Proof Complexity



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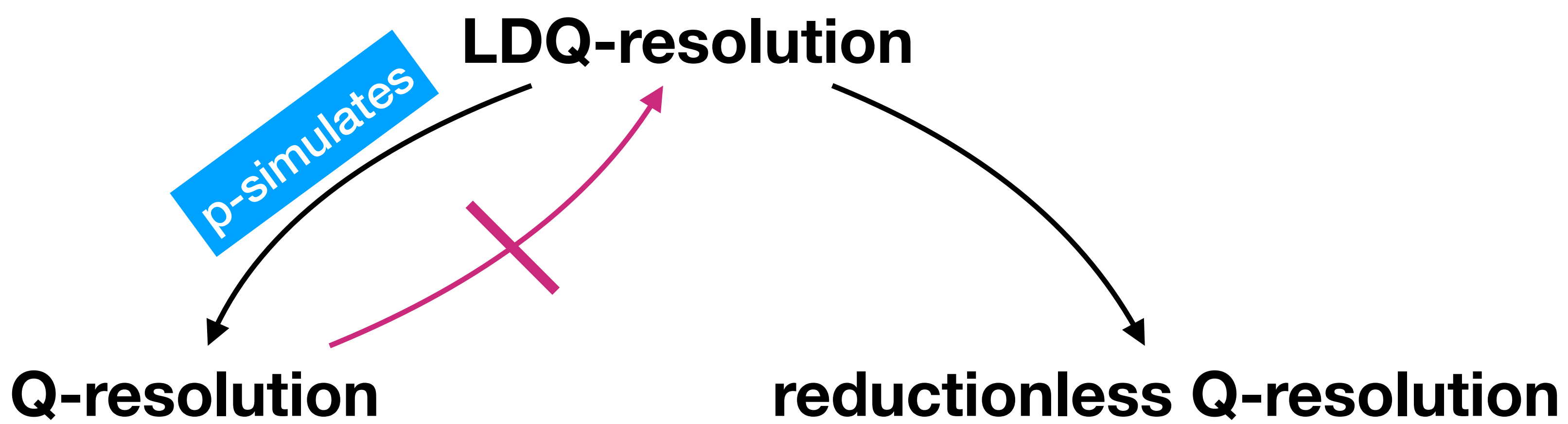
QParity has **linear** LDQ-resolution proofs.

Proof Complexity



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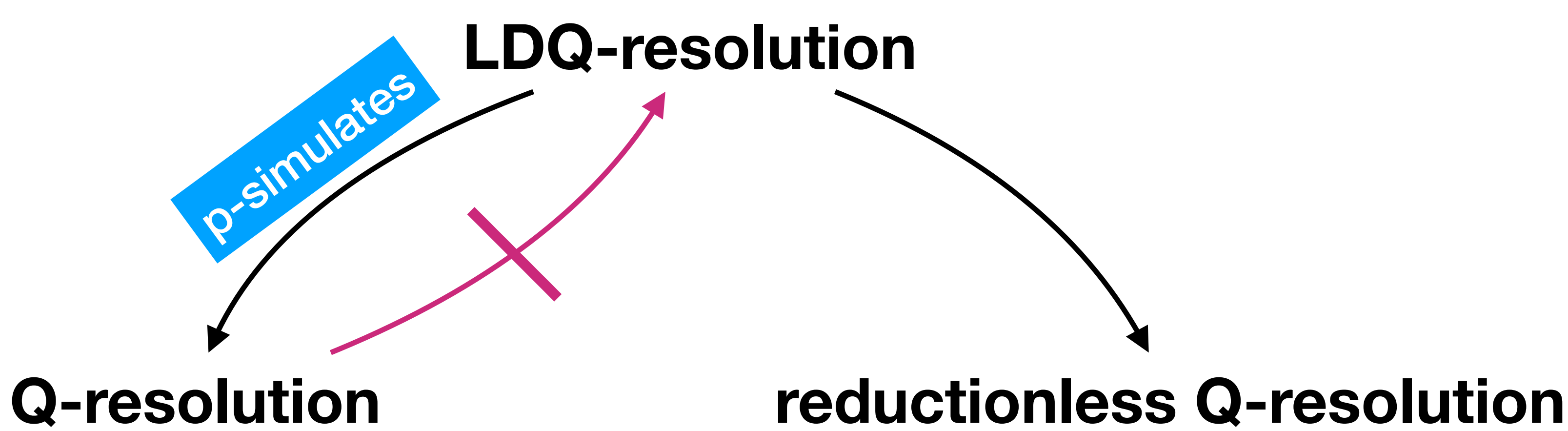
QParity has **linear** LDQ-resolution proofs. Chew'17

Proof Complexity



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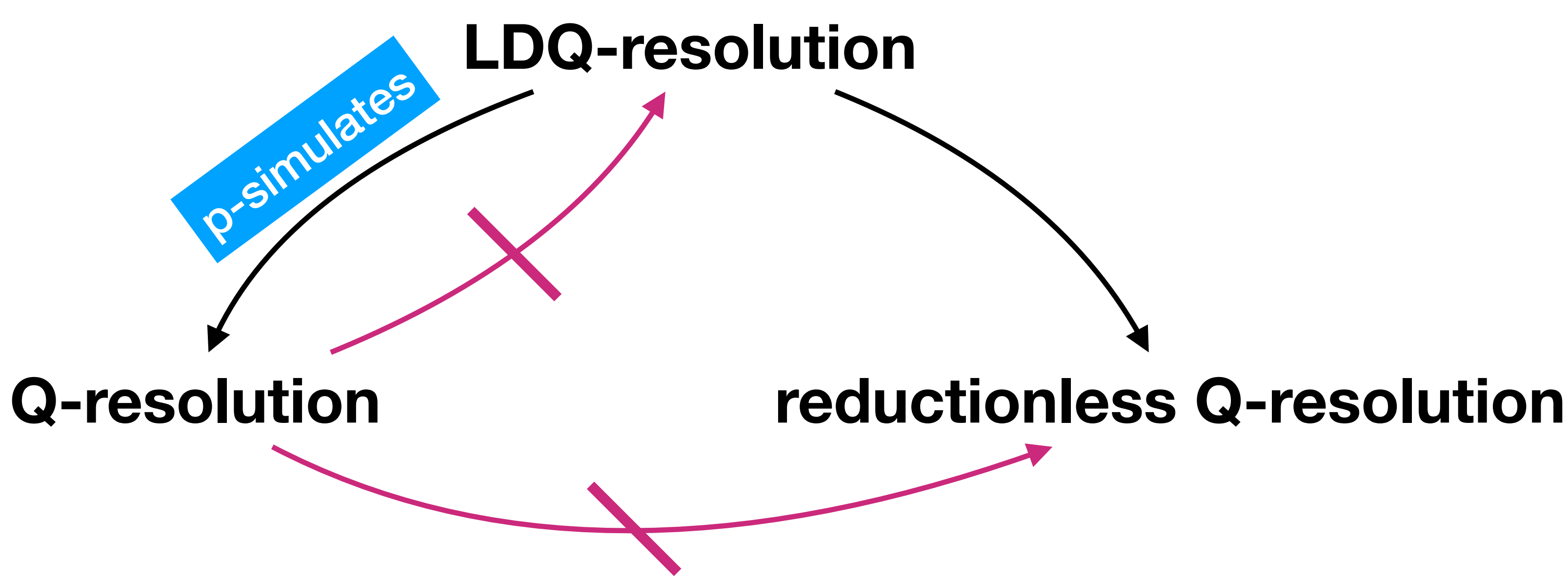
reductionless

Proof Complexity



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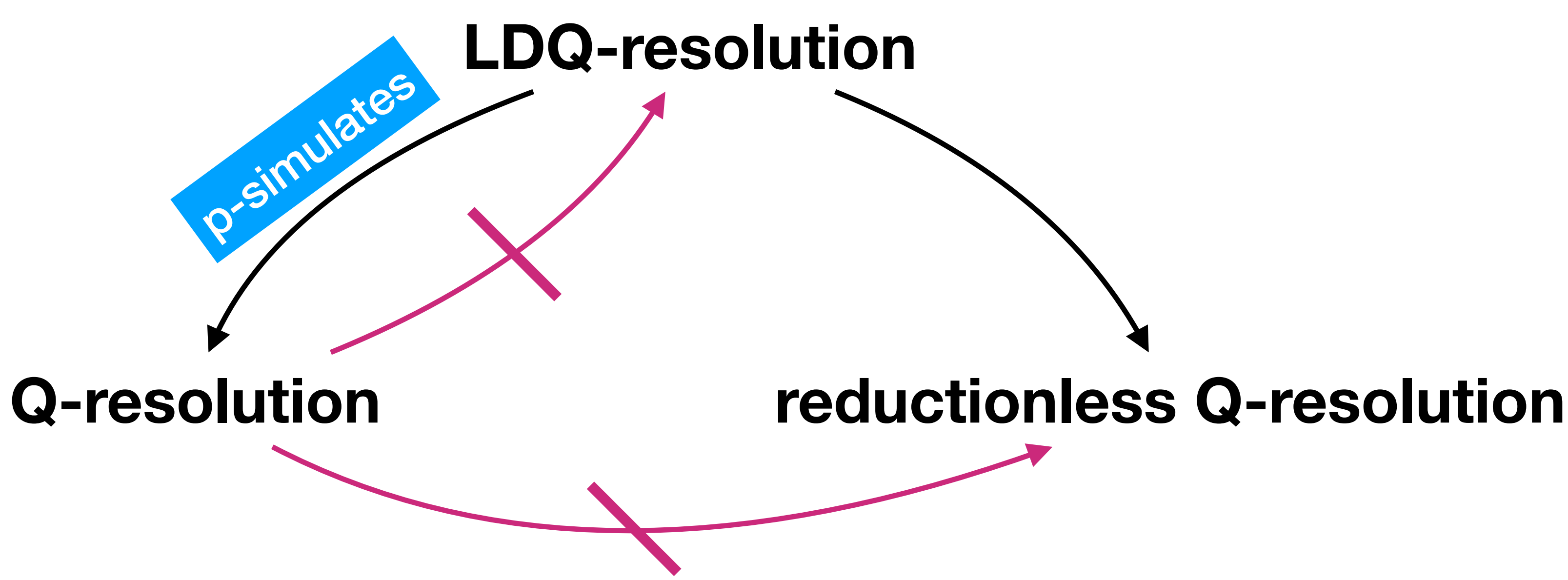
Proof Complexity



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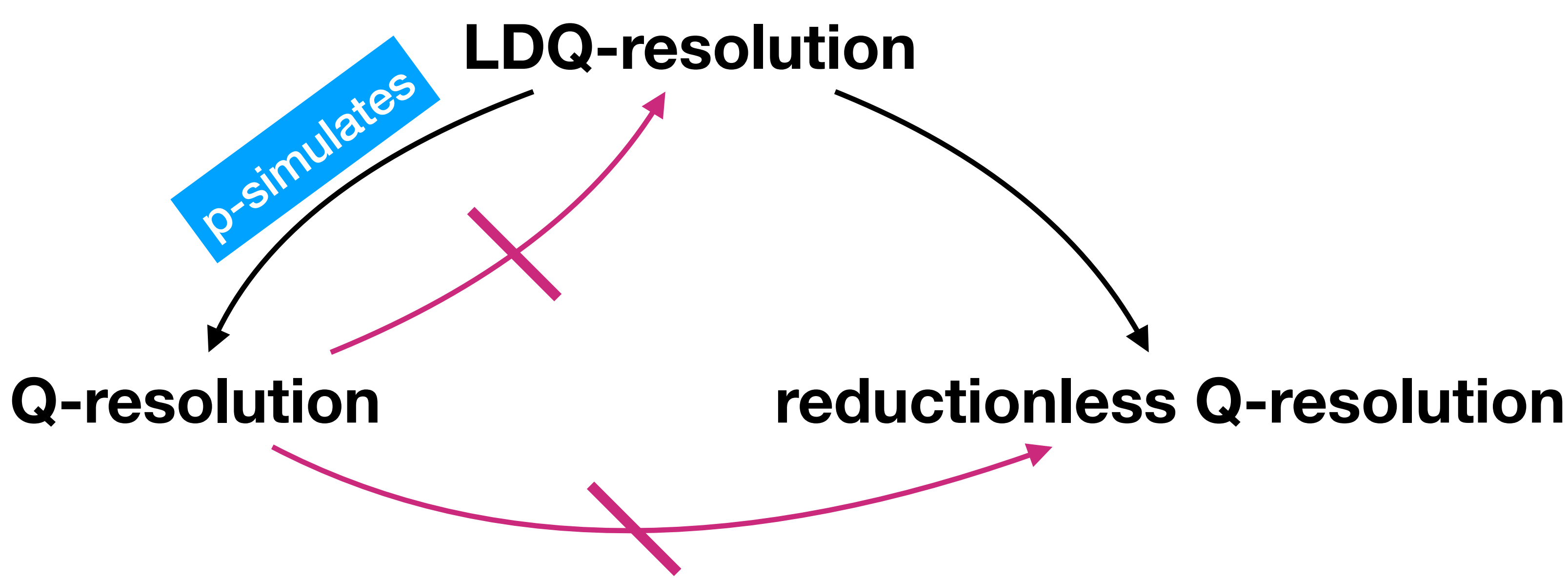
reductionless

Completion principle requires exponential reductionless Q-resolution proofs.

Proof Complexity



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reductionless

Peitl, S., Szeider

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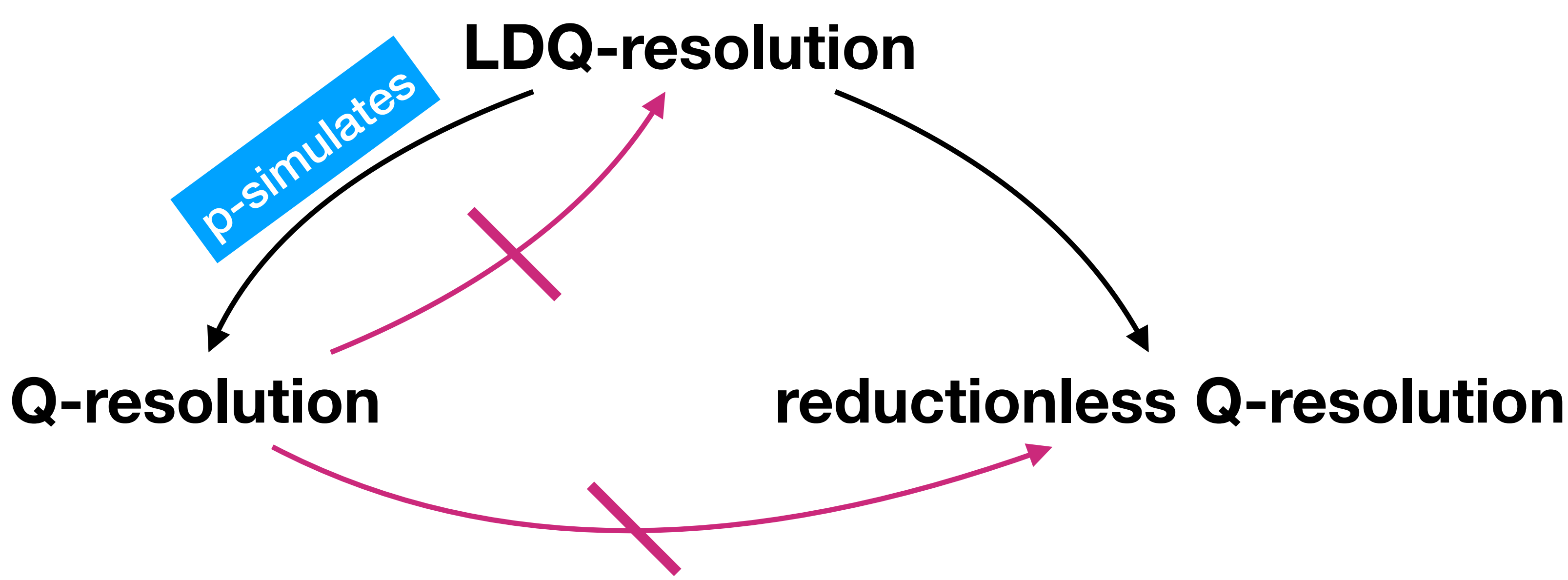
Proof Complexity



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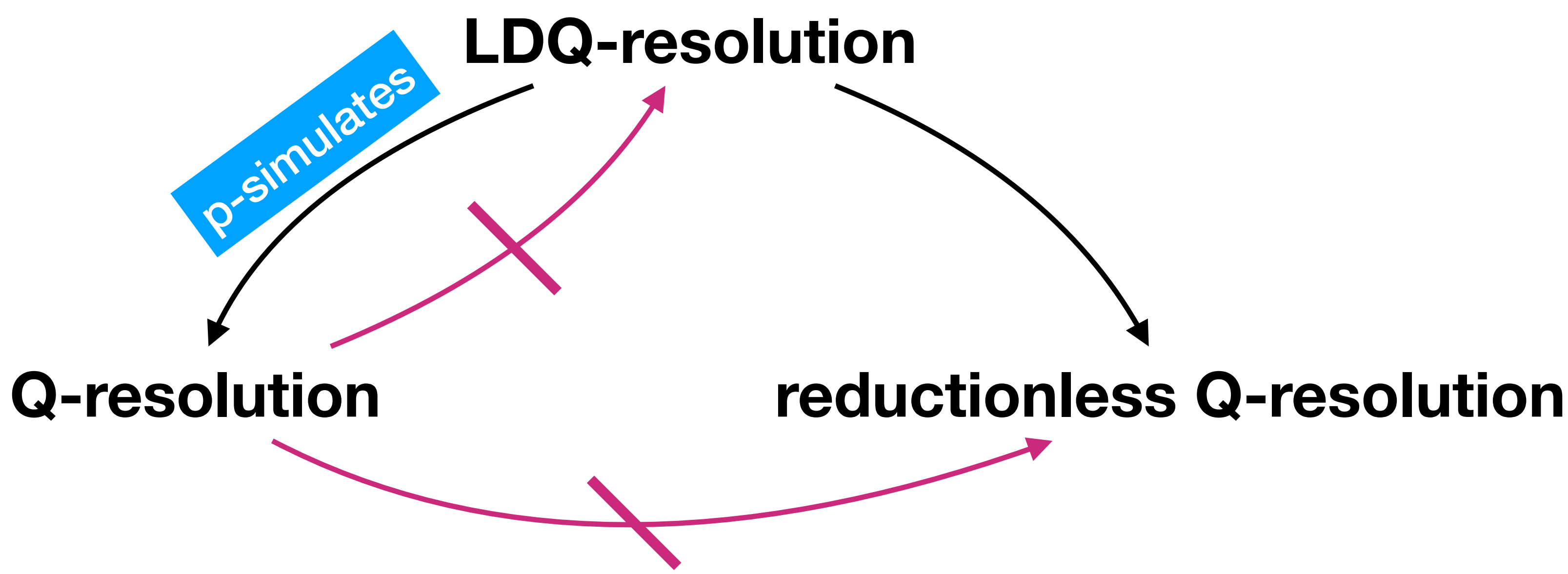
Completion principle requires exponential reductionless Q-resolution proofs.

Completion principle has linear (tree-like) Q-resolution proofs.

Proof Complexity



QParity does not have polynomial Q-resolution proofs. Beyersdorff, Chew, Janota'15

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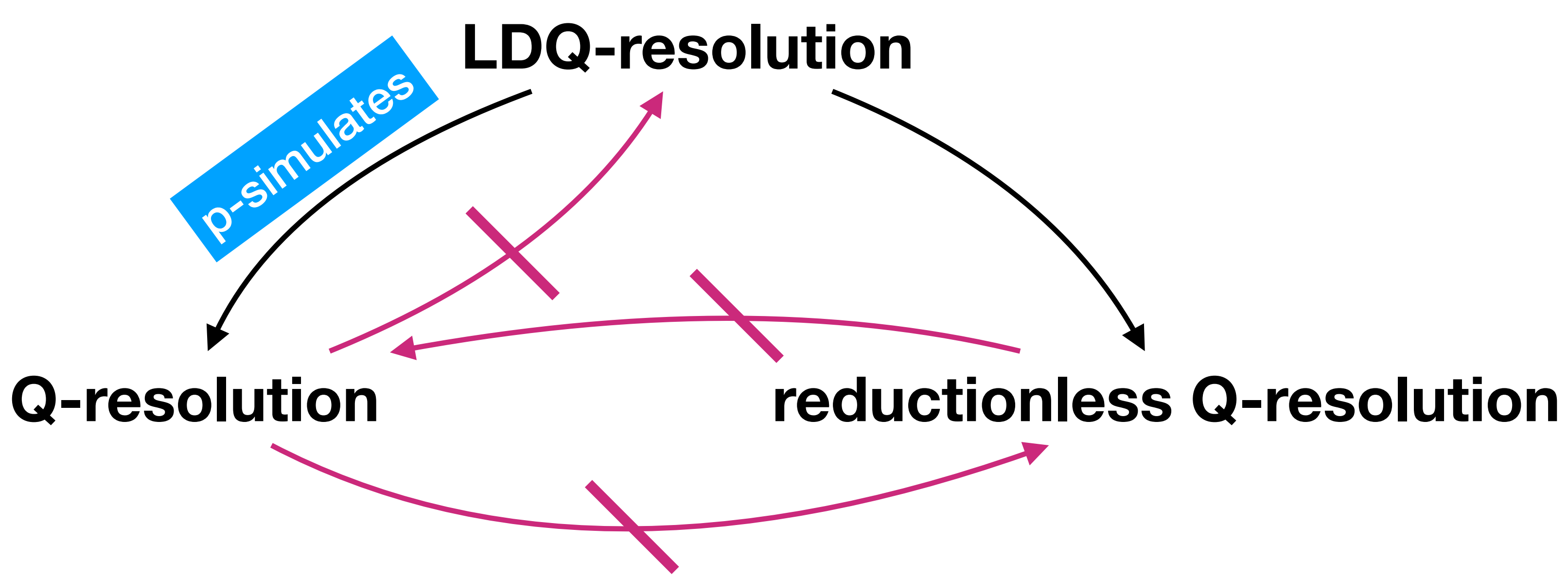
Proof Complexity



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reductionless

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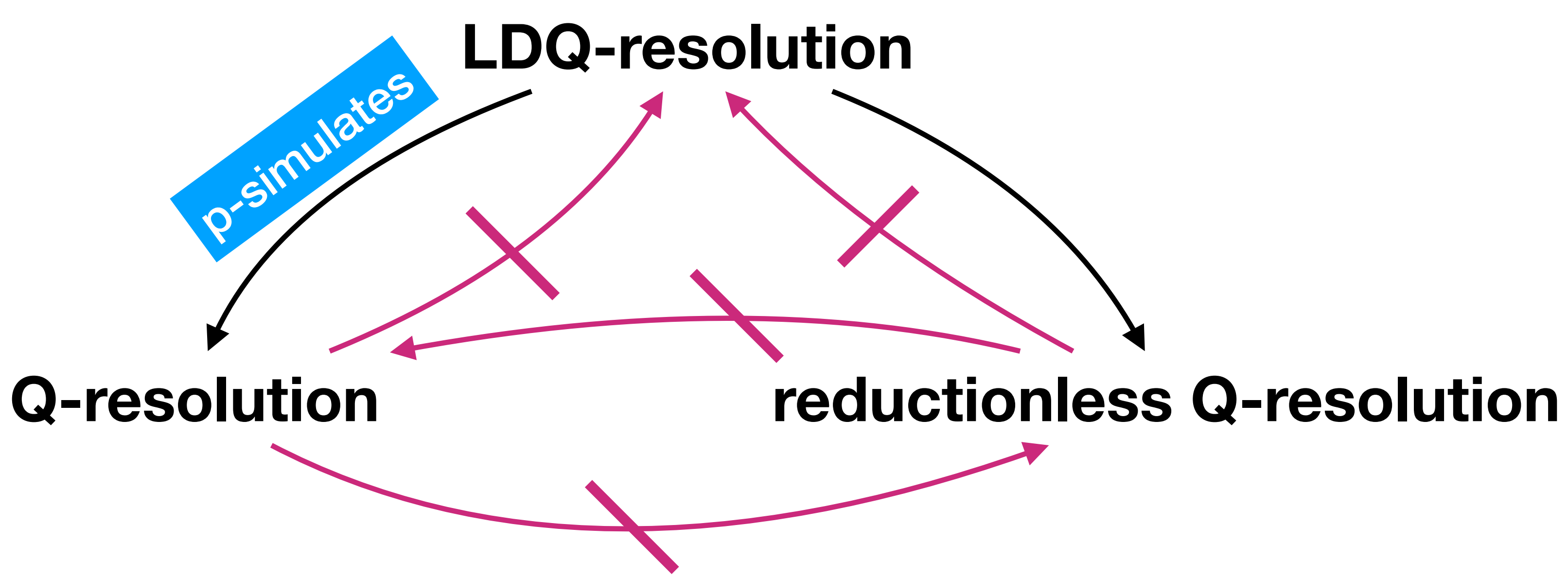

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Reductionless Q-resolution and Branching Programs

Reductionless Q-resolution and Branching Programs

Bjørner, Janota, Klieber '15

Reductionless Q-resolution and Branching Programs

Bjørner, Janota, Klieber ‘15

Beyersdorff, Blinkhorn, Mahajan ‘19

Reductionless Q-resolution and Branching Programs

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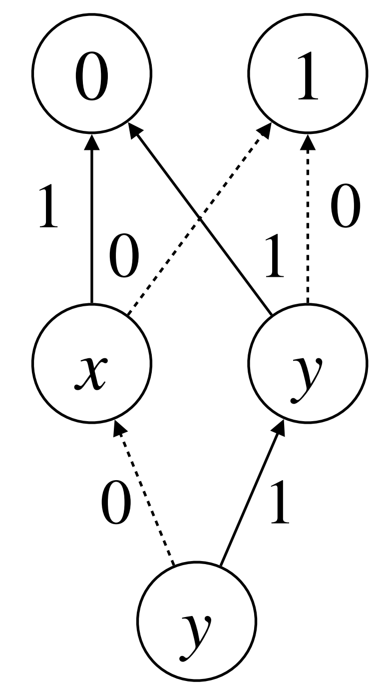
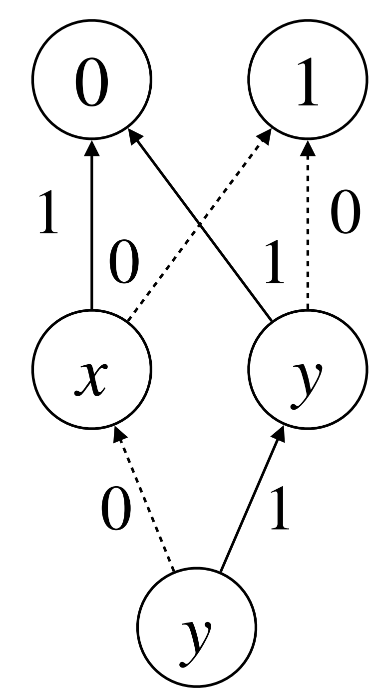


Reductionless Q-resolution and Branching Programs

Bjørner, Janota, Klieber '15

Beyersdorff, Blinkhorn, Mahajan '19

$$\frac{}{C \vee u}$$



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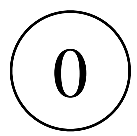

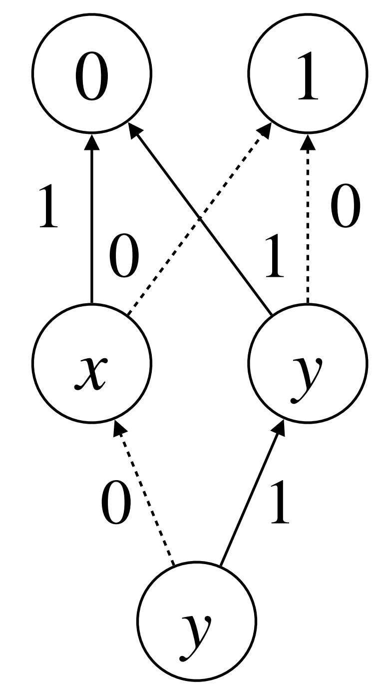

Reductionless Q-resolution and Branching Programs

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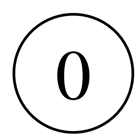
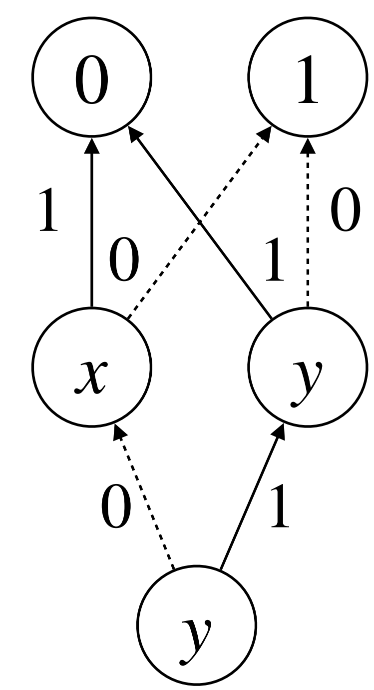


Beyersdorff, Blinkhorn, Mahajan ‘19

$$\frac{}{C \vee \neg u}$$



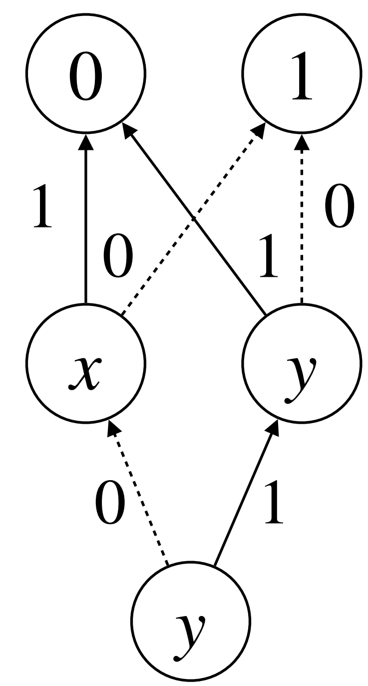
Reductionless Q-resolution and Branching Programs

Bjørner, Janota, Klieber ‘15

Beyersdorff, Blinkhorn, Mahajan ‘19

$\overline{C \vee u}$ 0

$\overline{C \vee \neg u}$ 1



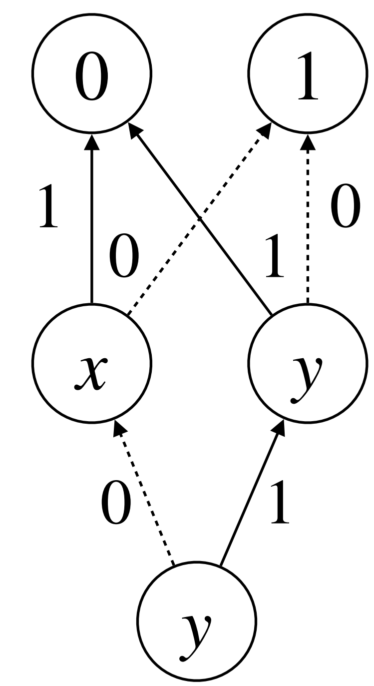
Reductionless Q-resolution and Branching Programs

Bjørner, Janota, Klieber '15

Beyersdorff, Blinkhorn, Mahajan '19

$$\frac{}{C \vee u} \quad (0)$$

$$\frac{}{C \vee \neg u} \quad (1)$$



$$e \vee u^* \vee C$$

Reductionless Q-resolution and Branching Programs

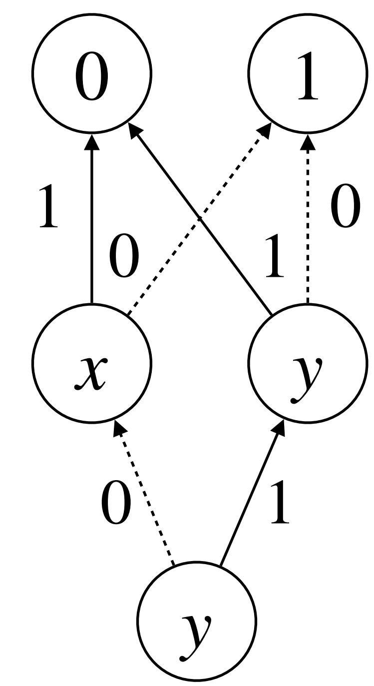
Bjørner, Janota, Klieber '15

Beyersdorff, Blinkhorn, Mahajan '19

$\overline{C \vee u}$ ⓪

$\overline{C \vee \neg u}$ ①

$e \vee u^* \vee C$



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$\overline{C \vee u}$ (0)

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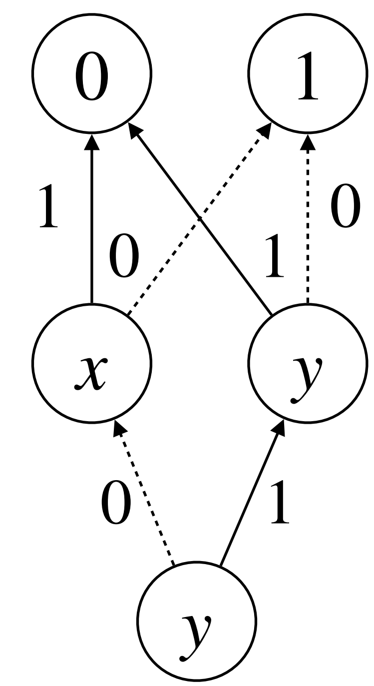

$e \vee u^* \vee C \quad \neg e \vee u^* \vee D$

Reductionless Q-resolution and Branching Programs

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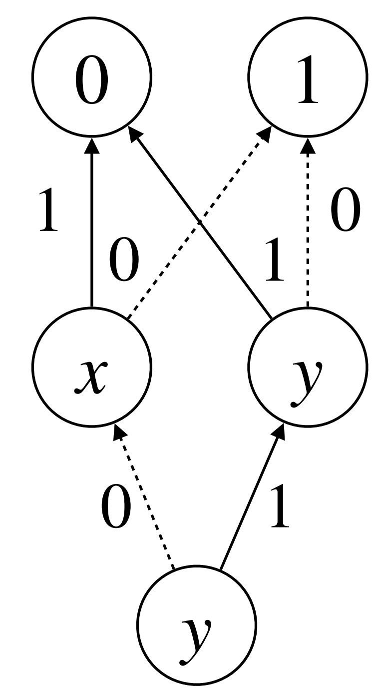

$e \vee u^* \vee C \qquad \neg e \vee u^* \vee D$

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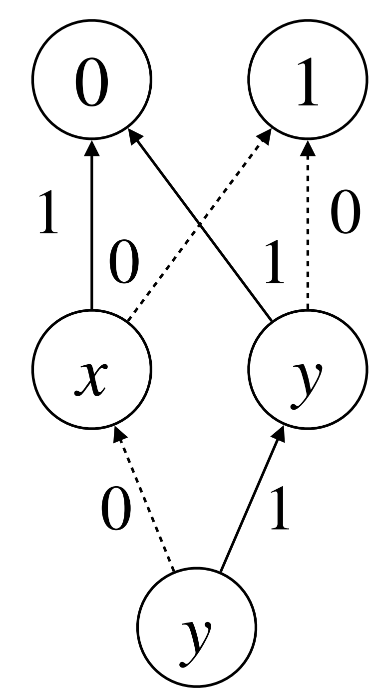
$$\overline{} \qquad \overline{}$$
$$C \vee u \quad (0) \qquad\qquad C \vee \neg u \quad (1)$$



$$\frac{e \vee u^* \vee C \qquad \neg e \vee u^* \vee D}{}$$

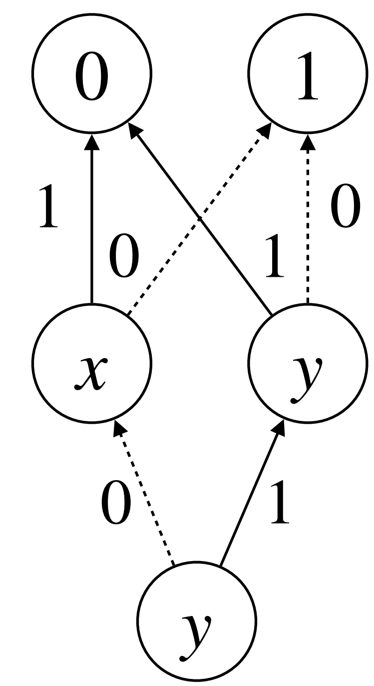
Reductionless Q-resolution and Branching Programs

Bjørner, Janota, Klieber ‘15

Beyersdorff, Blinkhorn, Mahajan ‘19

$$\frac{}{C \lor u} \quad \textcircled{0} \qquad\qquad \frac{}{C \lor \neg u} \quad \textcircled{1}$$

$$\frac{e \lor u^* \lor C \qquad \neg e \lor u^* \lor D}{C \lor D \lor u^*}$$



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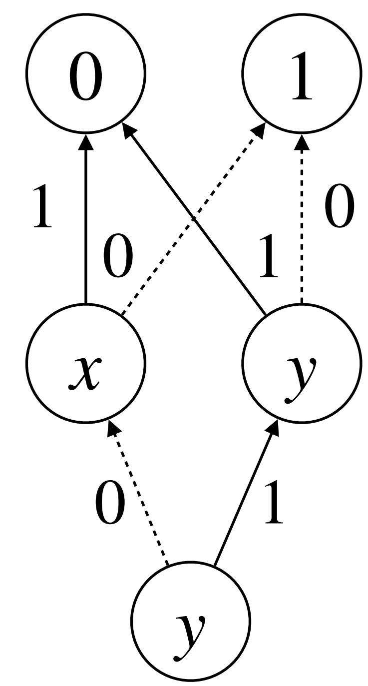
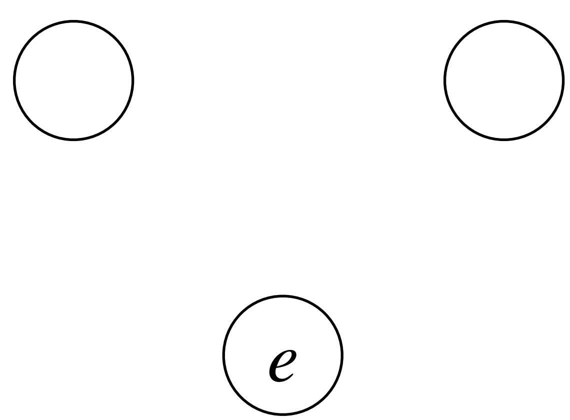


$$\frac{e \vee u^* \vee C \qquad \neg e \vee u^* \vee D}{C \vee D \vee u^*}$$



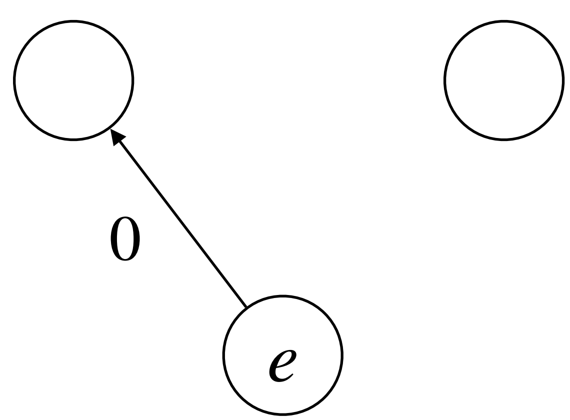
Reductionless Q-resolution and Branching Programs

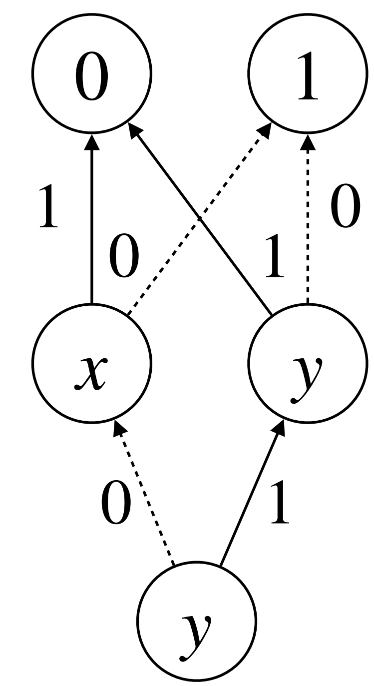
Bjørner, Janota, Klieber ‘15

Beyersdorff, Blinkhorn, Mahajan ‘19

$$\frac{}{C \vee u} \quad ⓪ \qquad\qquad \frac{}{C \vee \neg u} \quad ①$$

$$\frac{e \vee u^* \vee C \qquad \neg e \vee u^* \vee D}{C \vee D \vee u^*}$$





Reductionless Q-resolution and Branching Programs

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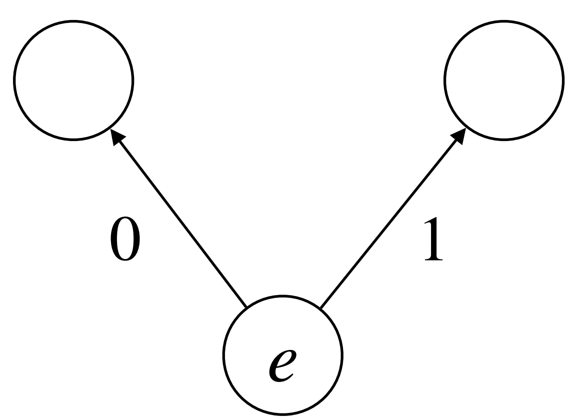

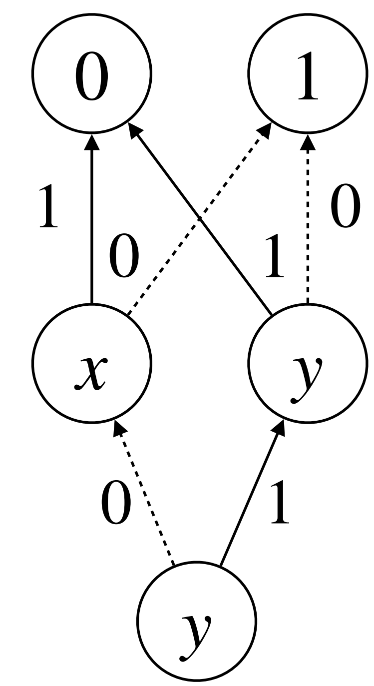

Semantic Lower Bounds for Restrictions of LDQ-Resolution

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Strategy functions corresponding to reductionless Q-resolution are **branching programs.**

Semantic Lower Bounds for Restrictions of LDQ-Resolution

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Regular reductionless Q-resolution

Semantic Lower Bounds for Restrictions of LDQ-Resolution

Strategy functions corresponding to reductionless Q-resolution are **branching programs.**

Regular reductionless Q-resolution ⇨

Semantic Lower Bounds for Restrictions of LDQ-Resolution

Strategy functions corresponding to reductionless Q-resolution are **branching programs.**

Regular reductionless Q-resolution ⇒ **read-once BPs**

Semantic Lower Bounds for Restrictions of LDQ-Resolution

Strategy functions corresponding to reductionless Q-resolution are **branching programs.**

Regular reductionless Q-resolution $\Rightarrow$ **read-once BPs**

$$\exists y_1 \dots \exists y_n \forall u \,.\, (u \nleftrightarrow f(y_1, \dots, y_n))$$

Semantic Lower Bounds for Restrictions of LDQ-Resolution

Strategy functions corresponding to reductionless Q-resolution are **branching programs.**

Regular reductionless Q-resolution ⇒ **read-once BPs**

hard for read-once BPs

$$\exists y_1 \ldots \exists y_n \forall u \, . \, (u \nleftrightarrow f(y_1, \ldots, y_n))$$

Semantic Lower Bounds for Restrictions of LDQ-Resolution

Strategy functions corresponding to reductionless Q-resolution are **branching programs.**

Regular reductionless Q-resolution ⇨ **read-once BPs**

hard for read-once BPs

$$\exists y_1 \ldots \exists y_n \forall u \,.\, (u \not\leftrightarrow f(y_1, \ldots, y_n))$$

tree-like LDQ-resolution

Semantic Lower Bounds for Restrictions of LDQ-Resolution

Strategy functions corresponding to reductionless Q-resolution are **branching programs.**

Regular reductionless Q-resolution ⇨ **read-once BPs**

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tree-like LDQ-resolution ⇨

Semantic Lower Bounds for Restrictions of LDQ-Resolution

Strategy functions corresponding to reductionless Q-resolution are **branching programs.**

Regular reductionless Q-resolution ⇒ **read-once BPs**

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Theorem

QParity does not have polynomial tree-like LDQ-resolution proofs.

Tree-like Long-Distance Q-resolution

Proof

C_1

C_2

C_3

...

C_{k-1}

$\square$

Strategy

$f_{u_1}, \ldots, f_{u_n}$

Tree-like Long-Distance Q-resolution

Balabanov, Jiang, Janota, Widl '15

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Proof → **Strategy**

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if $\neg C_3$ then $\neg$

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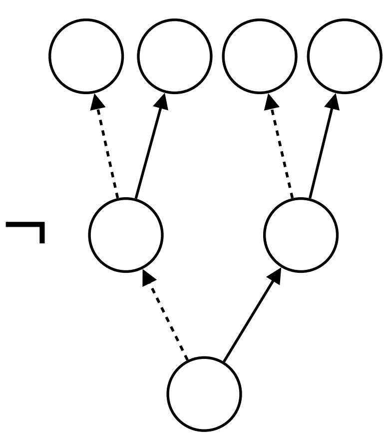

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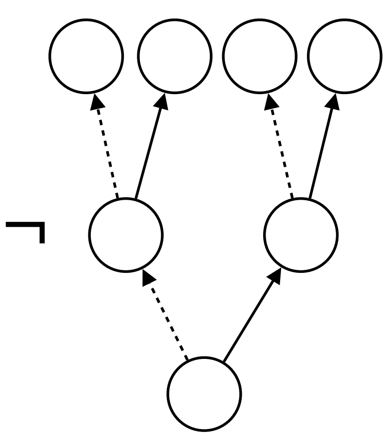

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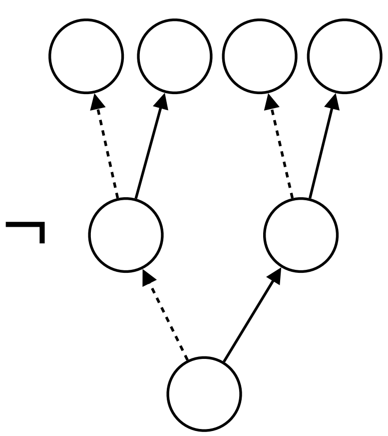

DT → CNF

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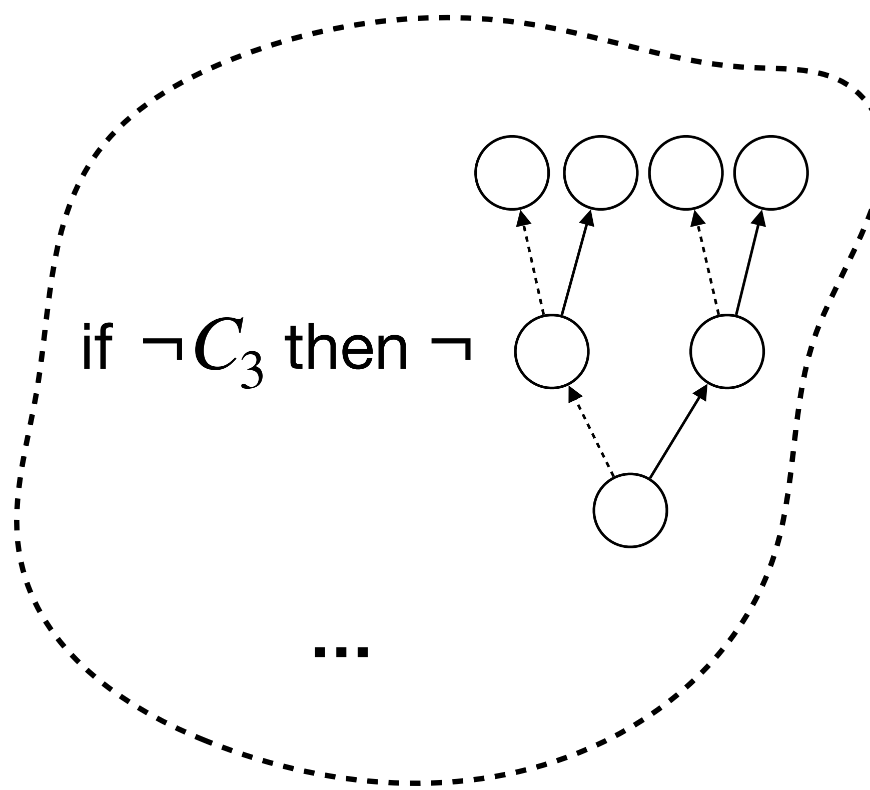

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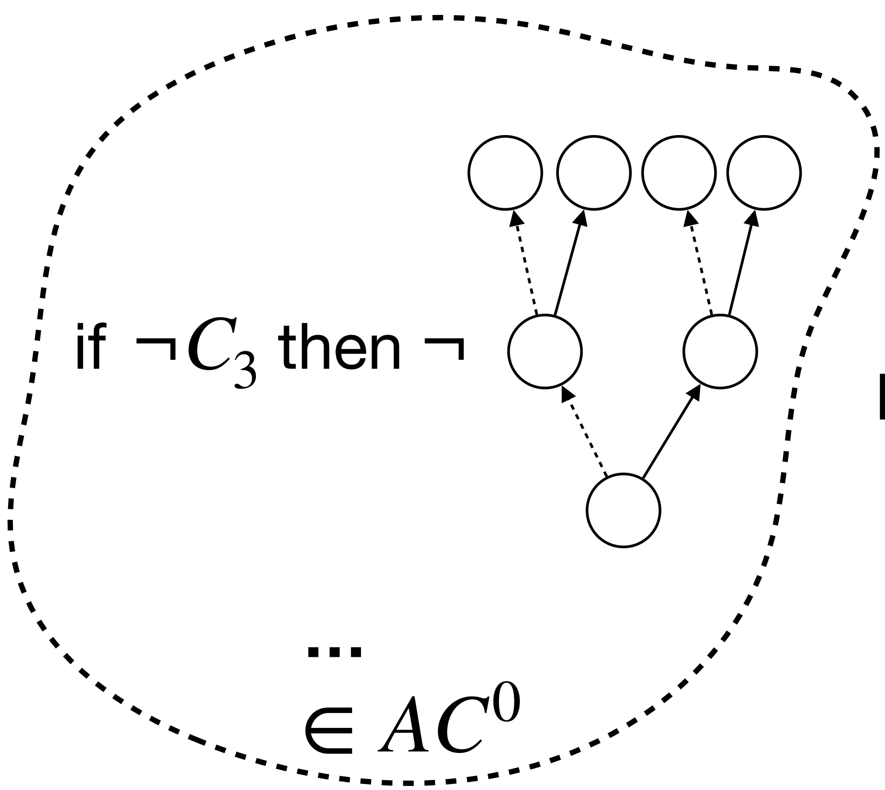

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Summary and Open Problems

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Semantic Lower Bound Techniques for LDQ-resolution